Stability

- Equilibria
- Dynamics & Stability
- Comparators
- Positive Feedback
- Schmitt Trigger
- Relaxation Oscillator
Do these two circuits really behave in the same way?
Op-Amp Dynamics

\[ V_+ - V_- = V_D \rightarrow V_d e^{j\omega t} \]

\[ V_{out} \rightarrow \frac{A V_d}{1 + j\omega RC} e^{j\omega t} \]

Texas Instruments \( \mu A741 \)
Amplifier Dynamics I

\[ V_{out} = AV_C \]

\[ RC \frac{dV_C}{dt} + V_C = V_+ - V_- \]

\[ V_+ = \frac{R_1}{R_1 + R_2} \]

\[ V_{out} = \delta V_{out} \quad (Positive \ Feedback) \]

\[ V_- = \frac{R_3}{R_3 + R_4} \]

\[ V_{out} = \delta V_{out} \quad (Negative \ Feedback) \]

\[ \frac{RC}{A} \frac{dV_{out}}{dt} + \left[ \frac{1}{A} + \delta - \delta \right] V_{out} = 0 \]

\[ \frac{dV_{out}}{dt} + \frac{A(\delta - \delta)}{RC} V_{out} = 0 \]
Amplifier Dynamics II

\[ \frac{d V_{\text{out}}}{dt} + \frac{1}{\tau} V_{\text{out}} = 0 \]

\[ \tau = \frac{RC}{A(\delta - \delta)} \]

\[ V_{\text{out}}(t) = V_{\text{out}}(0) e^{-t/\tau} \]

\[ \tau \begin{cases} 
  > 0 & \delta > \delta^+ \quad \text{Negative Feedback} \\
  = 0 & \delta = \delta^+ \quad \text{Neutral Feedback} \\
  < 0 & \delta < \delta^+ \quad \text{Positive Feedback} 
\end{cases} \]

\[ \text{Disturbance} \]

\[ V_{\text{out}}(t) \]

\[ t \]

\[ \begin{array}{c}
\tau < 0 \quad \text{Unstable} \\
\tau = 0 \quad \text{Neutral Stability} \\
\tau > 0 \quad \text{Stable}
\end{array} \]
Comparators

\[ V^+ \quad V^- \]

Positive Supply

Negative Supply

\[ V_{\text{OUT}} \]

\[ V^+ - V^- \]

\[ V_{\text{OUT}} \]

Fast

\[ 10-100 \frac{V}{\mu s} = \text{Slew Rate} \]
Op Amp Versus Comparator I

In 6.002, the device

\[ \text{[Diagram of an amplifier]} \]

represents an amplifier, usually having ideal properties. It can be used in both negative-feedback amplification (Op-Amp) applications, and positive-feedback comparator (Comparator) applications.
Op Amp Versus Comparator II

Op-Amp ⇒ Designed to be largely linear, and for use in negative feedback applications. Implemented with a few high-gain stages having modest band width.

Comparators ⇒ Designed to be very fast, and for use in open-loop applications or with positive feedback. Often implemented with many very fast stages having modest gain. May employ positive feedback for high-speed.
Schmitt Trigger I

Could interchange roles of $V_{IN}$ and $V_{REF}$.

$V^+ < V^- \equiv V_{REF}$  

... Fast ...

$V^+ > V^- \equiv V_{REF}$

$V_{OUT} = -V_S$

$V_{OUT} = -V_S$
Schmitt Trigger II

\(-V_s \rightarrow +V_s\) Transition:

\[ V_{OUT} \frac{R_A}{R_A + R_B} + V_{IN} \frac{R_B}{R_A + R_B} > V_{REF} \Rightarrow V_{IN} > \frac{R_A + R_B}{R_B} V_{REF} + \frac{R_A}{R_B} V_s \]

\(+V_s \rightarrow -V_s\) Transition:

\[ V_{OUT} \frac{R_A}{R_A + R_B} + V_{IN} \frac{R_B}{R_A + R_B} < V_{REF} \Rightarrow V_{IN} < \frac{R_A + R_B}{R_B} V_{REF} - \frac{R_A}{R_B} V_s \]
Schmitt Trigger Application

Digital Data → Noise → S.T. → Digital Data

Voltages

- $V_S$
- $\Delta + \delta$
- $\Delta$
- $\Delta - \delta$
- $-V_S$

Time
Oscillator

Voltages

\[
\begin{align*}
V_s & \quad V_s/2 & \quad 0 & \quad -V_s/2 & \quad -V_s \\
V_- & \quad V_+ & \quad V_{out} & \\
\end{align*}
\]

Time