Buck Converter

- Principles of operation
- Analysis via averaging
- Analysis via filtering
- Natural response
Buck Converter

- DC Supply
- Buck Converter
- Load

\[ V_{IN} \]

\[ V_C \]

\[ V_{IN} \]

\[ 0 \]

\[ DT \]

\[ T \]

\[ T + DT \]

\[ 2DT \]

- Digital electronics wish to operate at low voltage \((V_{out})\) to reduce losses and heat.
- Power is cheaper to deliver at high voltages due to wiring costs at high current.
- Buck converters bridge the high \(\rightarrow\) low voltage gap.
Average Analysis

* Assume cyclic continuous \((i > 0)\) operation.
* Define an average: \(<x(t)> = \frac{1}{T} \int_{0}^{T} x(s) \, ds\).

Inductor: \(V_s(t) - V_{out}(t) = L \frac{di}{dt}\)

\(<V_s(t)> - <V_{out}(t)> = \frac{L}{i} (i(t) - i(0)) \equiv 0\)

\(<V_{out}(t)> = D V_{IN}\)

Capacitor: \(i(t) - \frac{V_{out}(t)}{R} = C \frac{dn_{out}}{dt}\)

\(<i(t)> - <V_{out}(t)> = \frac{C}{R} (V_{out}(t) - V_{out}(0)) \equiv 0\)

\(<i(t)> = \frac{<V_{out}(t)>}{R} = \frac{D V_{IN}}{R}\)

Duty cycle \(D\) is used to control \(V_{out}\).
Ripple Analysis

Fast switching $\Rightarrow$ inductor current is approximately piecewise linear.

$$\Delta i \approx \frac{<V_{\text{out}}>-<V_{\text{in}}>}{L}$$

$$= \frac{D(1-D)T V_{\text{in}}}{L}$$

High-frequency inductor current passes largely through capacitor because $\frac{1}{\omega C} \ll R$.

$$\Delta Q \approx \frac{1}{2} \Delta i \frac{T}{2}$$

$$= \frac{D(1-D)T^2 V_{\text{in}}}{8L}$$

$$\Delta V_{\text{out}} = \frac{\Delta Q}{C}$$

$$= \frac{D(1-D)T^2 V_{\text{in}}}{8LC}$$
Buck Converter $\rightarrow$ Low-Pass Filter

Fourier decomposition of $V_s(t)$:

$$V_s(t) = D V_{IN} + \sum_{n=1}^{\infty} \frac{2 V_{IN} \sin(n\pi D)}{n\pi} \cos(n\omega_s t)$$

$\quad \omega_s T = 2\pi$

Filter each Fourier component separately and then recombine results to get complete response. Can do this for all signals: $V_{OUT}(t)$, $i(t)$, ....
Filter Analysis (Driven Response)

\[ \tilde{V}_{\text{out}} = \frac{(R \parallel \frac{1}{j\omega C}) \tilde{V}_{\text{in}}}{j\omega L + (R \parallel \frac{1}{j\omega C})} = \frac{\tilde{V}_{\text{in}}}{1 - \omega^2 LC + j\omega L \frac{1}{R}} = H(j\omega) \tilde{V}_{\text{in}} \]

\[ |H(j\omega)| = \frac{1}{\sqrt{1 - \omega^2 LC + j\omega L \frac{1}{R}}} \]

\[ H(j\omega) \approx \frac{\omega_0^2}{\omega^2} \]

\[ \log |H(j\omega)| \]

\[ \omega_0 = \frac{1}{\sqrt{LC}} \]

\[ Z_0 = \sqrt{\frac{L}{C}} \]

\[ Q = \frac{R}{Z_0} \]

\[ H(j\omega) \approx Q \]

\[ \omega_0 = \frac{1}{\sqrt{LC}} \]

Demo \( \Rightarrow \) \( L = 1 \text{ mH} \); \( C = 141 \mu\text{F} \); \( R = 50 \text{ } \Omega \)

\( \Rightarrow \omega_0 = 2.6 \frac{\text{rad}}{s} (424 \text{ Hz}) \); \( Z_0 = 2.7 \text{ } \Omega \); \( Q = 19 \)

But, \( R_{\text{Inductor}} = 0.5 \Omega \) \( \Rightarrow \) \( Q = \frac{Z_0}{R_{\text{Inductor}}} = \frac{2.7}{0.5} = 5.3 \rightarrow 4.2 \)
Filtering Interpretation

- At $\omega=0$, $H(j\omega) = 1 \Rightarrow \bar{V}_{out} = D\bar{V}_{in} = \langle V_s(t) \rangle$. Same result as before!

- $f_s = 40 \text{ kHz} \Rightarrow \omega_s = 80\pi \text{ krad/}\text{s} \Rightarrow \omega_0 = 2.6 \text{ krad/}\text{s}$. ⇒ All switching harmonics are greatly attenuated ⇒ Small ripple!

- Ripple fundamental $\Rightarrow \frac{2\bar{V}_{in} \sin(\pi D)}{\pi} \frac{\omega_0^2}{\omega_s^2}$ amplitude.

For $D = \frac{1}{2}$, amplitude $= \frac{2\bar{V}_{in}}{\pi} \frac{T^2}{4\pi^2 LC} = \frac{T^2 \bar{V}_{in}}{2\pi^3 LC} \approx \frac{T^2 \bar{V}_{in}}{64 \pi^2 LC}$

Compare with $\frac{1}{2} \frac{T^2 \bar{V}_{in}}{32 LC} = \frac{T^2 \bar{V}_{in}}{64 LC}$ from ripple analysis!
Natural Response

\[ L \begin{array}{c} \text{C} \\ \text{R} \end{array} \begin{array}{c} \text{V}_{\text{out}} \\ - \end{array} \]

\[ + \quad \frac{C}{R} \frac{d\text{V}_{\text{out}}}{dt} + \frac{\text{V}_{\text{out}}}{R} + \frac{1}{L} \int_{-\infty}^{t} \text{V}_{\text{out}} \, dt = 0 \]

\[ \frac{d^2 \text{V}_{\text{out}}}{dt^2} + \frac{1}{RC} \frac{d\text{V}_{\text{out}}}{dt} + \frac{1}{LC} \text{V}_{\text{out}} = 0 \]

\[ \omega_0^2 \frac{2\alpha}{\omega_0^2 - \omega^2} \]

\[ e^{\pm \alpha} \Rightarrow s = -\alpha \pm j \sqrt{\omega_0^2 - \omega^2} \]

\[ \alpha = -\alpha + j \omega_0 \]

Lightly Damped

Natural response will appear for:
- change in input voltage \( V_{\text{in}} \);
- change in duty cycle \( D \);
- change in load resistance \( R \).

Demo:

Load

Load Variation

Extra Damping

\[ \begin{array}{c} \text{1 mH} \\ 141 \mu F \\ 50 \Omega \end{array} \]

\[ 470 \mu F \]

\[ 1.2 \]

- Does not affect "average" behavior.
- Does not carry much ripple current because \( 1.2 \gg \frac{1}{\omega_0 470 \mu F} \).
- Greatly reduces the natural response \( Q \).