Circuits With Nonlinear Resistors

- Circuits With Nonlinear Resistors
- Nonlinear Resistor Example (Diode)

- Analysis Options:
  - Closed-Form (Difficult)
  - Numerical (Mention Only)
  - Graphical (Provides Intuition)
  - Piecewise Linear Approximation (Quick)
  - Linearized/Incremental (Later)
Circuits With One Nonlinear Resistor

One Nonlinear Resistor

Linear

Thevenin

Norton
Exponential Diode

\[ i_D = I_S \left( e^{\frac{V_D}{KT}} - 1 \right) \]

\[ \frac{KT}{q} \approx 26 \text{ mV} @ 300 \text{ K} \]

\[ I_S \approx 10^{-13} \text{ A} \ldots 10^{-9} \text{ A} \]

- Common in communication, power and lighting applications.
- Solar cells, LEDs, and many rectifiers and photodetectors are diodes.
Node Method

\[ \frac{V_D - V_{TH}}{R_{TH}} + I_S \left( e^{\frac{V_D}{KT/q}} - 1 \right) = 0 \]

No closed-form solution! Now what?

- Numerical Analysis
- Graphical Analysis
- Approximate (Piecewise-Linear) Analysis
- Linearized (Incremental) Analysis
Graphical (Load-Line) Analysis

Thevenin:

\[ V_D = V_{TH} - R_{TH} i_D \]

Diode:

\[ i_D = I_S \left( e^{\frac{V_D}{kT/q}} - 1 \right) \]
Exponential Diode II

Exponential

<table>
<thead>
<tr>
<th>$V_D$</th>
<th>$i_D$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.54 V</td>
<td>100 nA</td>
</tr>
<tr>
<td>0.60 V</td>
<td>1 mA</td>
</tr>
<tr>
<td>0.66 V</td>
<td>10 mA</td>
</tr>
<tr>
<td>0.72 V</td>
<td>100 mA</td>
</tr>
<tr>
<td>0.78 V</td>
<td>1 A</td>
</tr>
<tr>
<td>0.84 V</td>
<td>10 A</td>
</tr>
</tbody>
</table>

$I_S \approx 10^{-13} A$
Several Models

Exponential

Compromise

Ideal

Model selection depends on the viewpoint.
**Method Of Assumed States**

Model each diode as ideal with an optional voltage offset or series resistance.

- Diode = On/Off $\Rightarrow$ 2 States
- $N$ Diodes $\Rightarrow 2^N$ possible state combinations

For each state combination, analyze the network/circuit. Use the constraints on $V_D$ (On $\Rightarrow i_D \geq 0$) and $i_D$ (Off $\Rightarrow V_D \leq 0$) to determine the valid operating range for each state-combination analysis.

- Advantage: diode models become simple
- Disadvantage: $2^N$ analyses
Half-Wave Rectifier I

Diode = On \( (V_D = 0, i_D \geq 0) \)

\[ V_{IN} \]

\[ V_{OUT} = V_{IN} - 0.6V \]
\[ i_D = \frac{V_{IN} - 0.6V}{R} \geq 0 \]
\[ V_{IN} \geq 0.6V \]

Diode = Off \( (i_D = 0, V_D \leq 0) \)

\[ V_{IN} \]

\[ V_{OUT} = 0 \]
\[ V_D = V_{IN} - 0.6V \leq 0 \]
\[ V_{IN} \leq 0.6V \]
Half-Wave Rectifier II

![Graph showing the behavior of a half-wave rectifier with a diode being turned on and off.]

- **Diode Off**: When the diode is off, the output voltage `V_{out}` is zero for all input voltages `V_{in}`.
- **Diode On**: When the diode is on, the output voltage `V_{out}` equals the input voltage `V_{in}` for all input voltages above 0.6 V.

The graph illustrates the voltage `V_{in}` over time, with the diode being turned on and off to demonstrate the rectification process.