Massachusetts Institute of Technology  
Department of Electrical Engineering and Computer Science  

6.002 – Circuits & Electronics  
Fall 2018  

Quiz #1  

10 October 2018  

Name: ________________________________________________  

Recitation Time: 11  12  1  

- There are 15 pages in this quiz, including this cover page.  
- Please put your name in the space provided above, and circle the time of your recitation.  
- Please do not remove any pages from this quiz.  
- Do your work for each question within the boundaries of that question, or on the back of the preceding page. When finished, clearly indicate your answer, perhaps by circling it.  
- This is a closed-book quiz, but calculators and a single two-sided page of notes are allowed.  
- Good luck!  

<table>
<thead>
<tr>
<th>Problem 1</th>
<th>Problem 2</th>
<th>Problem 3</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Problem 4</th>
<th>Problem 5</th>
<th>Total Grade</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Problem 1: Network Discovery – 15%

Shown below is a network comprising one voltage source with an unknown value $V_S$ and two resistors with unknown resistances $R_1$ and $R_2$. Also shown below is the voltage-current relation measured at the port of the network. Finally, it is observed that resistors $R_1$ and $R_2$ in the network dissipate 4 mW in total when the port is left as an open circuit. Use the $v$-$i$ relation and the dissipated power to determine $V_S$, $R_1$ and $R_2$. 

![Network Diagram]

![Voltage-current Relation Diagram]
Problem 2: Resistor-Capacitor Networks – 30%

All parts of this problem concern the network shown below.

(2A) Prior to $t = 0$, the switch $S1$ in the network shown above is open, and the network reaches equilibrium. Determine the equilibrium capacitor voltage $V_c$ at $t = 0^-$. 

(2B) The switch S1 closes at $t = 0$. Determine the capacitor voltage $V_c$ at $t = 0^+$, just after the switch closes.
(2C) After the switch closes, as $t \to \infty$, the network again reaches equilibrium. Determine the new equilibrium capacitor voltage $V_c$. 
(2D) Given that switch S1 is closed, determine the effective resistance $R$ seen by the capacitor.
(2E) After switch S1 closes, the capacitor voltage evolves with a characteristic time constant $\tau$. Determine $\tau$. 
(2F) Derive an expression for $V_c(t)$ for $t \geq 0^+$. 
(2G) Sketch and label $V_c(t)$ for $t \geq 0^+$. 

\begin{center}
\begin{tikzpicture}[scale=1]
    \draw[->] (-0.5,0) -- (3.5,0) node[right] {$t$};
    \draw[->] (0,-0.5) -- (0,3.5) node[above] {$V_c(t)$};
    \draw (0,0) -- (3,0) node[below] {0};
    \draw (0,3) -- (0,0) node[left] {0};
\end{tikzpicture}
\end{center}
Problem 3: One Op Amp & Multiple Inputs – 15%

Determine $v_{\text{OUT}}$ as a function of the source voltages $v_0$, $v_1$ and $v_2$, and the circuit resistances $R_1$ through $R_4$. Assume that the op amp is ideal.
Problem 4: Equivalence – 15%

Find the Norton equivalent of the network shown below as seen from the indicated port.

![Network Diagram]
Problem 5: DAC – 25%

Your friend does not like the DAC we built in lab and proposes the alternative design shown below. The DAC output is $V_0$. Further, assume that the independent voltage sources for all bits can take on only the values of 0 V or $V_B$.

(5A) How many unique voltages can be generated by the newly-proposed DAC? Explain with supporting equations if you prefer. Please answer in terms of $N$, $R$ and $V_B$. 
(5B) Is the answer from Part (A) greater or less than the number of unique voltages one could generate using a R-2R DAC with an equivalent number of bits, as built in lab and shown below with four bits? Please briefly explain. The DAC output is again $V_O$, and the independent voltages sources for all bits can again take on only the values of 0 V or $V_B$. 

![Diagram of a R-2R DAC with four bits](image)
(5C) What is the smallest non-zero voltage, and the largest voltage, that the newly-proposed DAC can produce? Please answer in terms of $N$, $R$ and $V_B$. 
A four-bit version of the newly-proposed DAC is shown below. At its output port, derive a Thevenin equivalent for it as a function of $R$ and the voltage source values $V_1$ through $V_4$. 

![Diagram of the DAC circuit]