4. (20 points) Consider the circuit below:

![Circuit Diagram]

a) (10 points) Derive an expression for the control current $I_A$ in terms of $R_1$, $R_2$, $R_3$, $K$ and $v_{IN}$.

\[ I_A = \frac{V_{IN} - e_2}{R_3} \implies e_2 = V_{IN} - I_A R_3 \]

\[ I_A + K I_A = \frac{e_2}{R_2} \]

\[ I_A + K I_A = \frac{V_{IN} - I_A R_3}{R_2} \]

\[ R_2 I_A + K R_2 I_A = V_{IN} - I_A R_3 \]

\[ I_A (R_2 + K R_2 + R_3) = V_{IN} \]

\[ I_A = \frac{V_{IN}}{R_2 + K R_2 + R_3} \]
b) \((10\,\text{points})\) Derive an expression for the voltage \(v_0\) in terms of \(R_1, R_2, R_3, K\) and \(v_{IN}\) (but not \(I_A\)).

\[
\begin{align*}
    v_0 &= K I_A - R_1 + e_2 \\
    v_0 &= K I_A R_1 + v_{IN} - I_A R_3 \\
    v_0 &= I_A (R_1 K - R_3) + v_{IN}
\end{align*}
\]

From last problem:

\[
I_A = \frac{v_{IN}}{R_2 + K R_2 + R_3}
\]

\[
\therefore v_0 = v_{IN} \left(\frac{R_1 K - R_3}{R_2 + K R_2 + R_3}\right) + v_{IN}
\]

\[
v_0 = v_{IN} \left(\frac{R_1 K + R_2 + K R_2 + R_3}{R_2 + K R_2 + R_3}\right)
\]

\[
v_0 = \left(\frac{R_1 K + R_2 + K R_2}{R_2 + K R_2 + R_3}\right) v_{IN}
\]
The problem studies circuits involving independent sources, linear resistors and linear dependent sources. Note that the circuit in Part (A) is used again in Parts (B) and (C), though it is extended in the process.

(1A) Determine and draw the Thevenin equivalent for the circuit shown below as viewed from the indicated terminals. Numerical results with appropriate units are expected.

\[ e_1 = \frac{15}{45}(-15) = -5\text{ V} \]
\[ e_2 = \frac{30}{45}(+15) = 10\text{ V} \]
\[ V_{oc} = e_1 + e_2 = 5\text{ V} \]

\[ R_{TH} = \left(\frac{15}{1130}\right) + \left(\frac{15}{1130}\right) = 20 \Omega \]
A 20-Ω resistor is now connected to the circuit from Part (A) as shown below. What is the Thevenin equivalent of the modified circuit as viewed from the indicated terminals? Numerical results with appropriate are expected.

\[ V_{oc} = \frac{20}{40} (5V) = 2.5V \]

\[ R_{TH} = 20 \parallel 20 = 10 \text{ Ω} \]
The circuit from Part (A) is again modified, this time by the addition of a dependent source as shown below. What is the Thevenin equivalent of the modified circuit as viewed from the indicated terminals? *Numerical results with appropriate units are expected.*

\[ V_{oc} : \quad i' + 4i = 0 \]
\[ \therefore \quad i' = 0 \]
\[ V_{oc} = 5V \]

\[ I_{sc} : \quad i' = -\frac{5}{20} \text{A} \]

\[ 4i = -1 \text{A} \]
\[ I_{sc} = -i' - 4i = \frac{5}{4} \text{A} \]

\[ R_{th} = \frac{5V}{\frac{5}{4} \text{A}} = 4 \Omega \]
1. (16 points) Consider the circuit below:

![Circuit Diagram]

a) (10 points) Derive an algebraic expression for the current $I_1$ flowing through resistor $R_1$.

**Do by superposition.**

**Subcircuit a:**
(open $V_0$)

\[ I_{1a} = \frac{V_0}{R_1 + R_2} \]

**Subcircuit b:**
(short $V_0$)

\[ I_{1b} = I_0 \cdot \frac{R_1}{R_1 + R_2} \]

**All together**

\[ I_1 = I_{1a} + I_{1b} = \frac{V_0}{R_1 + R_2} + I_0 \cdot \frac{R_1}{R_1 + R_2} = \frac{V_0 + I_0 R_2}{R_1 + R_2} \]
b) (6 points) Calculate the power dissipated in the entire circuit if \( R_1 = R_2 = R_3 = 1 \, k\Omega \), \( V_0 = 1 \, V \) and \( I_0 = 1 \, mA \)?

Let us compute \( I_1 \):

\[
I_1 = \frac{1 \, V + 1 \, mA \times 1 \, k\Omega}{2 \, k\Omega} = 1 \, mA
\]

Then,

\[
W_1 = I_1^2 R_1 = (1 \, mA)^2 \times 1 \, k\Omega = 1 \, mW
\]

\[
W_2 = I_0^2 R_2 = (1 \, mA)^2 \times 1 \, k\Omega = 1 \, mW
\]

Since \( I_1 = I_0 \), there is no current through \( R_2 \) and

\[
W_2 = 0
\]

Power delivered by \( V_0 \):

\[
W_4 = V_0 I_1 = 1 \, V \times 1 \, mA = 1 \, mW
\]

Power delivered by \( I_0 \):

\[
W_5 = I_0 (V_0 - I_1 R_1 + I_0 R_3) = I_0 (1 - 1 \times 1 + 1 \times 1) = 1 \, mW
\]

The power delivered and absorbed add up, right.
3. (12 points) Derive and draw the Thevenin equivalent model of the circuit below as seen from Port A. Make sure to indicate the positive and negative terminals on the Thevenin equivalent.

Let us transform the circuit:

\[
\begin{align*}
V_0/R_3 & \quad \Rightarrow \quad R_3 \\
R_1 & \quad \Rightarrow \quad R_1 + \frac{R_2}{R_1 + R_2}
\end{align*}
\]

Then, the Thevenin resistance is:

\[
R_{TH} = \frac{1}{R_1 + (R_2 + R_3)}/R_1
\]

\[V_{TH} \approx V_{oc}, \text{ we have a voltage divider:}\]

\[
V_{TH} = \left(\frac{I_0 R_3 + V_s}{R_1}ight) \frac{R_1}{R_1 + R_2 + R_3}
\]
Problem 2:

\[ i_t = \frac{N_T - V_I}{R_1 + R_2} + \frac{V_T - AV_I}{R_3} \]

\[ u = V_I + \frac{R_1}{R_1 + R_2} (N_T - V_I) \]

\[ = \frac{N_T}{R_1 + R_2} - \frac{V_I}{R_1 + R_2} + \frac{N_T}{R_3} - \frac{AV_I}{R_3} = \frac{AR_1}{R_3(R_1 + R_2)} (N_T - V_I) \]

\[ R_{Th} = \frac{1}{\frac{1}{R_1 + R_2} + \frac{1}{R_3} - \frac{AR_1}{R_3(R_1 + R_2)}} = \frac{(R_1 + R_2)R_3}{R_1(1-A) + R_2 + R_3} \]

\[ V_{Th} = \left( \frac{1}{R_1 + R_2} + \frac{A}{R_3} - \frac{AR_1}{R_3(R_1 + R_2)} \right) \frac{R_2(R_1 + R_2)}{R_2 + R_1 + R_3 - AR_1} V_I \]

\[ = \frac{R_2 + A(R_1 + R_2) - AR_1}{R_1(1-A) + R_2 + R_3} V_I \]
Problem 2 (continued):

Consider $I_{sc}$,

$$(R_1 + R_2) i_1 = R_3 i_2 + A U$$

$$i_2 + i_1 = I_0$$

$$I_0 = i_1 + i_1 (R_1 + R_2 - AR_1) \frac{1}{R_3}$$

$$I_{sc} = i_1$$

So,

$$I_N = \frac{I_0 \cdot R_3}{R_3 + R_1 + R_2 - AR_1}$$

Use test source to find $R_{th}$,

$$i_T = \frac{NT - AU}{R_1 + R_2 + R_3}$$

$$v = NT - i_T R_1$$

$$R_{th} = \frac{R_1 (1-A) + R_2 + R_3}{(1-A)}$$
Problem 2 (continued):

b) $R_{1w} = R_{Th}$ from part B

c) $R_{out} = R_{Th}$ from part A
Problem 2: (25 points) This question concerns the circuit shown in Figure 3. In Parts (A) through (D), use Node 5 as the reference node.

![Circuit Diagram](image)

Figure 3: Circuit for Problem 2

(A) What is \( e_4 \), the Node voltage at Node 4?

\[
e_4 = \frac{V_B}{R_1}
\]

(B) (i) What is the current \( i_{R_1} \) into Node 2 through resistor \( R_1 \)?

\[
i_{R_1} = \frac{I_A}{R_1} = \frac{e_1 - e_2}{R_1}
\]

(ii) If \( e_2 \) is known, what is \( e_1 \) in terms of \( e_2 \)?

\[
\frac{e_1 - e_2}{R_1} = I_A \implies e_1 = e_2 + I_A R_1
\]

\[
e_1 = e_2 + I_A R_1
\]
Problem 3:  (30 points)

The graph from Figure 4 shows the measured behavior of Box A at the terminals a,b. The circuit from Figure 4 is the Norton equivalent circuit of Box B at the terminals c,d.

(A) Determine the short-circuit current of Box A; that is, what is $i_A$ when $v_A=0$?

$$i_A = -3\, A$$

(B) Determine the open-circuit voltage of Box B; that is, what is $v_B$ when $i_B=0$?

$$v_B = 2A \cdot 2\, \Omega$$

$$v_B = 4\, V$$
(C) The two boxes are connected as shown in Figure 5. Determine $i_A$, $i_B$, $v_A$, and $v_B$ with the two boxes connected.

Figure 5: Circuit for Problem 3(C)

\[ i_A = -i_B, \quad v_A = v_B \]

Method 1 - Analytical

\[ i_A = -3A + \frac{1}{1\Omega} \cdot v_A * from \ graph \]

\[ i_B + 2A - \frac{v_B}{2\Omega} = 0 \Rightarrow i_B = -2A + \frac{1}{2\Omega} \cdot v_B * from \ Norton \ circuit \]

\[ i_A = -i_B, \quad v_A = v_B \]

\[-3A + \frac{1}{1\Omega} \cdot v_A = 2A - \frac{1}{2\Omega} \cdot v_A \Rightarrow v_A = (\frac{1}{1\Omega} + \frac{1}{2\Omega})^{-1} \cdot 5A \]

\[ v_A = \frac{2}{3} \Omega \cdot 5A = \frac{10}{3} \, V \]

\[ v_B = \frac{10}{3} \, V \]

\[ i_A = -3A + \frac{1}{1\Omega} \cdot \frac{10}{3} \, V \]

\[ i_A = -\frac{1}{3} \, A \]

\[ i_B = -\frac{1}{3} \, A \]

\[ v_A = \frac{10}{3} \, V \]

\[ v_B = \frac{10}{3} \, V \]
Method 2 - Combine circuits

Norton of A:

\[ i_A = \frac{10}{3} \Omega (2A + i_B) \]

\[ i_B = -\frac{1}{3} A \]

\[ i_A = -i_B = \frac{1}{3} A \]

Method 3 - Graphical

\[ i_B = -i_A, \quad v_A = v_B \rightarrow \text{draw both on } i_A, v_A \]
(D) Consider the terminal pair p,q. They are defined below with the boxes connected as in Figure 6.

Figure 6: Circuit for Problem 3(D)

Draw the Thévenin or Norton equivalent circuit seen at the terminals p and q. Also, write an expression for \( i \) in terms of \( v \).

\[
\text{Easiest as Norton}
\]

\[
\begin{align*}
3A & \quad 3Ω & \quad 2Ω & \quad 2A & \Rightarrow & \quad 5A & \quad \frac{5}{3}Ω \\
\end{align*}
\]

\[
\begin{align*}
\nu &= \frac{2}{3}Ω \cdot (5A + i) \\
i &= \frac{\nu}{\frac{2}{3}Ω} - 5A = \frac{3}{2} \nu - 5 \\
i &= \frac{3}{2} \nu - 5
\end{align*}
\]
Problem 8

\[ G_1(e_1-V_1) + G_2(e_1-e_2) - I_2 = 0 \]
\[ G_2(e_2-e_1) + G_3(e_2) + I_1 + G_4(e_2-e_3) = 0 \]
\[ I_2 + G_4(e_3-e_2) + G_5(e_3+V_2) = 0 \]
For Parts (1E) and (1F) consider the network shown below. It contains two nodes at which the voltages are unknown. Those voltages are labeled as $e_1$ and $e_2$.

(1E) By carrying out a node analysis, write two algebraic equations that can be solved for the two unknown node voltages $e_1$ and $e_2$ in terms of the source values $V$ and $I$, and the resistances of the four resistors. Do not solve the equations for $e_1$ and $e_2$.

\[
\begin{align*}
\frac{e_1 - V}{R_1} + \frac{e_1}{R_2} + \frac{e_1 - e_2}{R_3} &= 0 \\
\frac{e_2 - e_1}{R_3} + \frac{e_2}{R_4} + I &= 0
\end{align*}
\]

\[
\begin{align*}
e_1 \left[ \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} \right] + e_2 \left[ \frac{-1}{R_3} \right] &= V \left[ \frac{1}{R_1} \right] \\
e_1 \left[ \frac{-1}{R_3} \right] + e_2 \left[ \frac{1}{R_3} + \frac{1}{R_4} \right] &= -I
\end{align*}
\]
(1F) Assume that $R_1 = R_2 = 2R$ and that $R_3 = R_4 = R$. Determine the node voltage $e_1$ in terms of $V$, $I$ and $R$.

Use superposition

$$e_1 = \frac{V}{3}$$

$$e_1 = -\frac{I}{3} \cdot R$$

$$e_1 = \frac{1}{3} (V - RI)$$
3. (40 points) Consider the circuit below:

![Circuit Diagram]

a) (8 points) Calculate the power delivered by the current source (numerical value and correct units expected).

The voltage across the current source, $V_i = 2 \text{ V}$, and the current thru it is 0.5 A.

$$P_i = iV = (0.5 \text{ A})(2 \text{ V}) = 1 \text{ W}$$

Following the convention that positive current flows into the positive terminal:

$$P_+ = (-0.5 \text{ A})(+2 \text{ V}) = -1 \text{ W}$$

and -1 W dissipated is +1 W delivered.

$$W_i = 1 \text{ W delivered}$$
b) (8 points) Calculate the power delivered by the voltage source (numerical value and correct units expected).

Combining the resistors: $\frac{R_2}{2} + \frac{R_3}{2} = 1\ \Omega$ in series with $R_1 = 1\ \Omega$ gives an equivalent circuit:

The current thru the $2\ \Omega$ resistor is $\dot{I}_{\text{req}} = \frac{2V}{2\ \Omega} = 1\ \text{A}$

$\dot{I}_T = \dot{I}_V + \dot{I}_{\text{req}} \Rightarrow \dot{I}_V = \dot{I}_T - \dot{I}_{\text{req}} = 0.5\ \text{A} - 1\ \text{A} = -0.5\ \text{A}$

So there is a $0.5\ \text{A}$ current leaving the voltage source.

$P_V = \dot{I}_V \cdot V_V = (0.5\ \text{A}) (2\ \text{V}) = -1\ \text{W}$ dissipated $\Rightarrow +1\ \text{W}$ delivered

$W = 1\ \text{W}$ delivered

c) (8 points) Calculate the power dissipated in resistor $R_1$ (numerical value and correct units expected).

Total current thru the $1\ \Omega$ $R_1$ is $\dot{I}_{\text{req}}$ from (b)

$\dot{I}_{R_1} = \frac{2V}{2\ \Omega} = 1\ \text{A}$

$P_{R_1} = (\dot{I}_{R_1}^2) \cdot R_1 = (1\ \text{A})^2 (1\ \Omega) = 1\ \text{W}$

$W_1 = 1\ \text{W}$
d) (8 points) Calculate the power dissipated in resistor $R_2$ (*numerical value and correct units expected*).

The current through $R_1$ is split evenly between $R_2$ and $R_3$.

\[ i_{R_2} = i_{R_3} = \frac{i_{R_1}}{2} = \frac{1}{2} A \]

\[ P_{R_2} = (i_{R_2})^2 R_2 = \left(\frac{1}{2} A\right)^2 (2 \Omega) = \frac{1}{2} W \]

\[ W_2 = \frac{1}{2} W \]

e) (8 points) Calculate the power dissipated in resistor $R_3$ (*numerical value and correct units expected*).

Same as $R_2$

\[ W_3 = \frac{1}{2} W \]