Problem 3: Integrator – 25%

Consider the circuit shown above with switch S1 *initially open* and the source is a constant 5V. Assume that the Op-Amp is ideal.

(3A) **6 pts** Find $v_{OUT}(t)$.

The open switch means the capacitor is disconnected from the circuit and no current can flow into or out of it. Furthermore, the disconnected node can float to any potential, thus the capacitive voltage drop cannot constrain the circuit potential in any way. It thus can be completely ignored in this part of the circuit. The remaining circuit is a straight-forward inverting op-amp with gain $\frac{100 \Omega}{1 \text{k}\Omega} = 0.1$ thus $v_{OUT} = -0.1v_{IN} = -0.5V$. Students commonly lost credit for neglecting to provide units or providing an incorrect sign.

$\boxed{v_{OUT}(t) = -0.5V}$
(3B) **6 pts** Suppose at $t = 0$, switch S1 is moved from an open to a closed position. Assume that the capacitor voltage $v_C(0) = 0$ (i.e. there is no initial charge). Find the time constant $\tau$ of the system after this event, i.e. for $t > 0$.  

Because the negative op-amp input acts as a virtual ground, the only impedance seen by the capacitor is the $100 \, \Omega$ resistor in parallel with it. Thus $\tau = RC = 100 \, \Omega \times 100 \, \text{nF} = 100 \, \Omega \times 10^{-9} \, F = 10 \times 10^{-6} \, \Omega F = 10 \, \mu s$. 

$$\tau = 10 \, \mu s$$
(3C) 7 pts Solve for $v_{\text{OUT}}(t)$ in the situation described in problem (3B) for $t > 0$. Leave $\tau$ as a symbol, rather than substituting your answer from (3B).

The capacitor starts out uncharged, and we know that in the absence of infinite currents, the capacitor voltage must be continuous, thus with the switch suddenly closed this means that $v_{\text{OUT}}(t = 0 + \epsilon) = v_{\text{OUT}}(t = 0 - \epsilon) = 0 \text{V}$. Meanwhile, at $t = \infty$ we know that steady state must be reached thus the capacitor can be treated as an open circuit and thus we have the same solution as for part A, meaning $v_{\text{OUT}} = -0.5 \text{V}$. Using the standard formula for 1st-order step response of circuits that approach a steady state, $v(t) = (v_{\text{init}} - v_{\text{final}})e^{\Delta t/\tau} - v_{\text{final}}$, this means that $v_{\text{OUT}}(t > 0) = -0.5 \text{V} + 0.5Ve^{-t/\tau}$.
(3D) **6 pts** At \( t = \tau \), assume the switch is switched *back to being open*. Solve for \( v_{\text{OUT}}(t) \) for \( t > \tau \).

As in part A, the capacitor can play no part in the circuit, the circuit output immediately steps to \(-0.5V\).
Problem 17: 4 points

Figure 16.

Derive an expression for \( v_o(t) \) in terms of \( v_1(t) \) and \( v_2(t) \).

First, superposition at the \( n^+ \) node:

\[
\begin{align*}
\sum^+ &= \frac{R/2}{R/2 + 2R} \cdot V_2 + \frac{2/3R}{R + 3/3R} \cdot V_2 \\
\sum^+ &= \frac{1}{5} V_2 + \frac{2}{5} V_2
\end{align*}
\]

In addition, \( \sum^- = \frac{7}{15} \cdot V_0 \Rightarrow \sum^- = \frac{5}{2} \cdot \sum^- \)

since \( \sum^- = \sum^+ \),

\[
\sum^- = \frac{5}{2} \left( \frac{1}{5} V_1 + \frac{2}{5} V_2 \right)
\]

\[
v_o(t) = \frac{V_1}{2} + \frac{V_2}{2}
\]
Problem 2 – 25%

This problem studies the op-amp circuit shown below. *Note that its output voltage is the voltage $v_{\text{OUT}}$. Assume that the op-amp is ideal. Also, assume that the circuit is stable. That is, assume that all voltages are finite for a finite input voltage $v_{\text{IN}}$.*. 

![Op-amp circuit diagram](image-url)
Determine the Thevenin-equivalent resistance of the op-amp circuit at its output port as a function of the circuit parameters. Note that the output port is the port at which $v_{out}$ is defined.

Let $v_{in} = 0$ and inject a test current into the positive output terminal.

\[ v_+ = v_- = v_{out} \quad \text{ideal op-amp} \]

\[ v_{amp} = v_- \frac{R_1 + R_2}{R_1} \quad \text{voltage divider} \]

\[ i_{test} = \frac{v_{out}}{R} + \frac{v_{out} - v_{amp}}{R} \quad \text{KCL at positive} \]

Substitutions $\Rightarrow v_{out} = \frac{R \cdot R_1}{R_1 - R_2} \cdot i_{test}$

Thevenin Resistance $R_{TH}$
(2B) Determine the Thevenin-equivalent voltage and the Norton-equivalent current of the op-amp circuit at its output port as a function of the circuit parameters.

\[ V_+ = V_- = V_{\text{out}} \quad \text{... ideal op-amp} \]

\[ V_{\text{out}} - V_{\text{in}} = \frac{V_{\text{out}}}{R_1} \quad \text{... voltage divider from } V_{\text{out}} \text{ to } V_- \]

\[ \frac{V_{\text{out}} - V_{\text{in}}}{R} + \frac{V_{\text{out}} - V_{\text{amp}}}{R} = 0 \quad \text{... KCL at positive \ V_{\text{out}} \ terminal} \]

Substitutions \Rightarrow \ V_{\text{out}} = \frac{V_{\text{in}} R_1}{R_1 - R_2} \quad \text{Thevenin voltage } V_{TH}

\[
\text{Norton Current} = \frac{V_{TH}}{R_{TH}} = \frac{V_{\text{in}}}{R} = I_N
\]
(2C) The op-amp circuit is connected to a load resistor at its output port as shown below. Assume that \( R_1 = R_2 = R \). Determine the current \( i_L \) into the load as a function of \( v_{IN} \), \( R \) and \( R_L \).

\[
\text{Op - Amp Circuit}
\]

\[
\begin{array}{c}
\text{As } R_1 = R_2 \rightarrow R, \quad V_{TH} \rightarrow \infty, \quad R_{TH} \rightarrow \infty \\
\text{and } i_N = \frac{V_{IN}}{R}.
\end{array}
\]

The op-amp circuit becomes a current source.

\[
i_L = \frac{V_{IN}}{R}
\]
(2D) Assume that the op-amp output voltage $v_{\text{AMP}}$ must satisfy $|v_{\text{AMP}}| < V_S$ for the op-amp to operate properly. Still assuming that the op-amp circuit is connected to a load resistor as shown in Part 2C, and still assuming that $R_1 = R_2 = R$, determine the allowable range of $R_L$ that ensures proper operation of the circuit. Do so in terms of $v_{\text{IN}}$, $R$ and $V_S$.

$$N_{\text{OUT}} = \frac{N_{\text{IN}}}{2} \cdot R_L \quad \text{... Current source into load resistance}$$

$$v_{\text{AMP}} = 2 \cdot N_{\text{OUT}} \quad \text{... Voltage division from } N_{\text{AMP}} \text{ to } V_-$$

$$\left| \frac{2 \cdot N_{\text{IN}} \cdot R_L}{R} \right| < V_S$$

$$0 < R_L < \frac{V_S \cdot R}{2 \cdot |N_{\text{IN}}|} \quad \text{... } R > 0 \text{ and } V_S > 0$$
(2E) The load resistor in Part 2C is replaced by a load capacitor having capacitance $C$. Assume that $v_{IN}$ is constant, and that $v_{OUT}(t=0)=0$. Still assuming $R_1 = R_2 = R$, determine $v_{OUT}(t)$ for $t > 0$ as a function of $v_{IN}$, $R$ and $C$.

\[ i_N = C \frac{dN_{OUT}}{dt} = \frac{v_{IN}}{R} \]

\[ N_{OUT} = \frac{1}{RC} \int_{0}^{t} v_{IN} \, dt = \frac{t}{RC} N_{IN} \]
The load resistor in Part 2C is replaced by a load inductor having inductance $L$. Assume that $v_{IN}$ has been zero for all time prior to $t = 0$, and that $v_{IN} = Kt$ thereafter. Still assuming $R_1 = R_2 = R$, determine $v_{OUT}(t)$ for $t > 0$ as a function of $v_{IN}$, $R$, $L$ and the constant $K$.

$$v_{OUT} = L \frac{di_N}{dt} = \frac{KL}{R}$$