11/07/18 Recitation Week 10.

* Active Filters.

1. Passive 1st order LPF:

\[
\frac{V_{\text{out}}}{V_{\text{in}}} = \frac{1}{j\omega C} \rightleftharpoons R + \frac{1}{j\omega C} = \frac{1}{1 + j\omega RC}
\]

\[
\omega_c = \frac{1}{RC}
\]

Now consider load:

\[
\frac{V_{\text{out}}}{V_{\text{in}}} = \frac{1}{j\omega C \parallel R_L} \rightleftharpoons R + \frac{1}{j\omega C \parallel R_L}
\]

\[
\omega_c = \frac{1}{RC} (1 + \frac{R}{R_L})
\]

The load resistance influences the cut-off frequency.

2. Solution: Use a voltage buffer:

\[
\frac{V_{\text{out}}}{V_{\text{in}}} = \frac{1}{1 + j\omega RC}, \text{ regardless of load resistance.}
\]

(The input impedance of Op-Amp is very large!)
You can get a gain (amplify the voltage) at the same time filtering.

\[ \frac{V_o}{V_{IN}} = \frac{1 + \frac{R_1}{R_2}}{1 + j\omega RC} \cdot \frac{1}{1 + \frac{1}{j\omega RC}} \]

One issue: The voltage source resistance influences the \( \omega_c \)!

\[ \omega_c = \frac{1}{(R + R_S)C} \]

The type of source you choose shifts the filter performance!

3. An alternative design: Put the reactive component in the feedback loop!

\[ \frac{V_{OUT}}{V_{IN}} = ? \]

\[ \frac{V_o - V_i}{Z} = \frac{V_i}{R_2} \]

\[ V_o = \left( \frac{Z}{R_2} + 1 \right) V_i = \left( \frac{R_1}{R_2} \frac{1}{j\omega C} + 1 \right) V_i \]
\[
\frac{V_0}{V_i} = 1 + \frac{R_i}{j\omega C} = 1 + \frac{R_i/R_2}{1 + j\omega R_1 C}
\]

**DC gain:** \((\omega = 0)\) \(\frac{V_0}{V_i} = 1 + \frac{R_i}{R_2}\)

If \(R_i >> R_2\), \(\frac{V_0}{V_i} \approx \frac{R_i}{R_2}\), \(\omega_c \approx \frac{1}{R_1 C}\).

Note that cut-off frequency only depends on component values \((R_1, R_2, C)\) inside the filter, not on source/load impedance.

4. Can you realize a LPF with an inverting-Amp topology?

   ![Inverting Amplifier Diagram]

   How to analyze the circuit?

   The same way as pure resistive circuit:

   \[RCL\text{ at node } V_\omega: \frac{V_i - 0}{R_1} = \frac{0 - V_\omega}{R_2/R_1} = \frac{1}{R_2/j\omega C}\]

   \(\text{(impedance of parallel circuit)}\)

   \[\omega_c = \frac{1}{R_2 C}\]

   DC gain: \(\frac{R_2}{R_1}\), cut-off frequency \(\omega_c = \frac{1}{R_2 C}\)

   **In 1st order filter**

   \[\log|H| \rightarrow \frac{\alpha}{\omega} \leftarrow \text{Can we make the slope steeper? more ideal LPF.}\]

   \[\omega_c \rightarrow \log \omega\]
5. 2nd order filter:

1) Cascading two 1st order filters:

\[
\frac{V_0}{V_i} = \frac{V_o}{V_a} \cdot \frac{V_a}{V_i} = \left( \frac{R_2}{R_1} \right)^2 \frac{1}{1+j\omega CR_2} \cdot \frac{1}{\omega^2}
\]

for \( \omega \gg \frac{1}{R_2C} \).

Disadvantage: Two Op-Amp; high cost, take large space.

2) Realize a 2nd-order filter with one op-amp?

Let's analyze:

\[
\frac{e-V_0}{R_2} = \frac{V_o}{j\omega C_2} \quad \Rightarrow \quad e = (j\omega R_2 C_2 + 1) V_o.
\]

Node \( V_+ \):

\[
\frac{e-V_0}{R_2} + \frac{V_0-V_+}{R_2} + \frac{V_+}{j\omega C_1} = 0.
\]

With 1 and 2:

\[
\frac{1}{V_0} = \frac{1}{R_1 R_2 C_2} \frac{1}{(j\omega)^2 + j\omega \left( \frac{1}{R_1 C_1} + \frac{1}{R_2 C_1} \right) + \frac{1}{R_1 R_2 C_2}}.
\]

DC gain: 1, \( \omega \to \infty \), \( |V_0| \propto \frac{1}{\omega^2} \).
cut-off frequency: \( w_0 = \frac{1}{\sqrt{R_1 R_2 C_1 C_2}} \) (in analogy to \( \frac{1}{2C} \))

quality factor: \( Q = \frac{w_0}{2\alpha} \)

\[ 2\alpha = \frac{1}{R_1 C_1} + \frac{1}{R_2 C_1} \]

One can realize a 2nd order LPF similar to the LRC circuit, but only with \( R \) and \( C \).

Sallen-Key 2nd order filter!

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