MOSFETs: A Followup

Let's remember how a resistor would work with similar labels:

We can drop resistors into circuits such as voltage dividers like shown here:

And if we made one of our resistors a function of an outside phenomenon such as:
- Light-sensitivity (photo resistor)
- Sound-sensitivity (microphone)
- Pressure-sensitivity (pressure resistor)
- Temperature (thermistor)
- ...and many more

This means the V-I relationship has a second “input” (not just V but also the physical phenomenon)...so you'll need to maybe draw multiple curves!

We could have a circuit that generates a voltage as a function of the physical phenomenon. This is basically how we convert signals of one form into another:
The purpose of a transistor is to give us the ability to make one electrical signal be the
function of another electrical signal

The thermistor above had a "third terminal" in the form of the fact that outside heat can
flow in and impact its V-I curve

For a transistor we are just making that "third terminal" be an actual explicit connection

with me so far? hopefully

The trickiness of MOSFETS (and transistors in general) is that the way in which their V-I
curves change based on that third voltage isn't nice and linear

AND (unfortunately or fortunately depending on how you want to approach it/how late at night
it is/how close to an exam we are) the way the V-I curve changes is also based on what the V
is (vDS in our labeling) (so it is non-linear from that angle as well). The result is you get
somewhat complicated-looking curves:

\[ i_D = \begin{cases} 
0 & \text{if } V_{GS} < V_T \\
\frac{K}{2} (V_{GS} - V_T)^2 & \text{if } V_{DS} \geq V_{GS} V_T \text{ and } V_{GS} > V_T \\
K (V_{GS} - V_T - \frac{V_{DS}}{2}) V_{DS} & \text{if } V_{DS} < V_{GS} V_T \text{ and } V_{GS} > V_T 
\end{cases} \]
There are a few variables on the previous page which are new. We know $v_{GS}$ and $v_{DS}$. What about:

- $K$: This is an inherent property of the MOSFET based on how it was built. It will be given. Basically impacts scaling on the plots.
- $V_T$: This is the THRESHOLD VOLTAGE...this is the voltage at which point the MOSFET will "turn on" if $v_{GS}$ is above it.

The three cases shown on the previous page describe the three operating modes. The first one:

$$i_D = 0 \quad \text{if} \quad v_{GS} < V_T$$

is the "OFF" state of the MOSFET...it doesn't conduct current...it looks like an infinite home resistor. It isn't really drawn on the plot.

Then there are then two "ON" states for the MOSFET, each with different flavors:

The first one is the "Saturation" region:

$$i_D = \frac{K}{2} (v_{GS} - V_T)^2 \quad \text{if} \quad v_{DS} \geq v_{GS} - V_T \quad \text{and} \quad v_{GS} > V_T$$

Here current is based only on one input: $v_{GS}$. From the perspective of $v_{DS}$, the MOSFET is acting like a current source!!! Look at its V-I curve below...in the saturation region the V-I curves are flat! Just like a current source would be (constant current for all voltages in that area!)

Since the MOSFET acts like a current source in saturation we could always present to draw it like a current source as well!
The other "ON" mode of the MOSFET is the Triode Region

\[ i_D = k (v_{gs} - V_T) v_{ds} \quad \text{if} \quad v_{ds} < v_{gs} - V_T \quad \text{and} \quad v_{gs} > V_T \]

Here we can see that the current **IS** a function of \( v_{DS} \) which means as we vary \( v_{DS} \) in our plots, we should see some variation in our current as we vary \( v_{DS} \). In fact if we expand out our expression:

\[ i_D \approx k (v_{gs} - V_T) v_{ds} - \frac{k v_{ds}^2}{2} \]

since we'll only be using this equation when \( v_{DS} \) is rather small (< \( v_{GS} - V_T \)), we can ignore the second term (square of a small term is even smaller) and this turns into (-ish):

\[ i_D \approx k (v_{gs} - V_T) v_{ds} \]

and if we plot this out as a V-I curve (\( v_{DS} \) vs. \( i_D \)):

\[ v_{gs} = 5V \]
\[ v_{gs} = 4V \]
\[ v_{gs} = 3V \]
\[ v_{gs} = 2V \]

In this region it sort of looks as if for differing \( v_{GS} \) values you get different effective resistances (straight V-I curves) and yeah that's sort of right. That's in fact why this region is sometimes called the "Ohmic Region". When in the triode region we can say:

\[ i_D \approx \frac{1}{R_{on}} v_{ds} \quad R_{on} \propto \frac{1}{k(v_{gs} - V_T)} \]

More correctly since we'll be in triode region until \( v_{DS} = a \) given \( v_{GS} - V_T \), that means the current at the point when we switch into saturation will be:

\[ i_D = \frac{k}{2} (v_{GS} - V_T)^2 \]

If we draw a line from \( v_{DS} = 0, i_D = 0 \) to \( v_{DS} = v_{GS} - V_T, i_D = \frac{k}{2} (v_{GS} - V_T)^2 \) we'll be able to get a line that is described by the expression

\[ i_D = \frac{k (v_{gs} - V_T)}{2} v_{ds} \quad \text{so} \quad R_{on} \propto \frac{1}{k(v_{gs} - V_T)} \]
What this means is that in the triode region the MOSFET will act like a voltage-dependent resistor with an ON resistance of about:

$$R_{on} \approx \frac{R}{K(v_{gs}-V_t)}$$

So we can redraw that plot sort of like the following:

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**FLOWCHART:**

In this class we won’t worry about figuring out what mode the MOSFET goes into...we’ll either know from the type of circuit we’re working with or tell you.

- **Saturation**
  - $V_{gs} > V_T$
  - $V_{ds} > V_{gs} - V_T$
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  - $V_{ds} > V_{gs} - V_T$
  - $V_{ds} > V_{gs} - V_T$
Let's analyze this simple circuit now:

For a given $V_{GS}$ what will $v_{out}$ be?

$v_{out}$ is just $v_{DS}$ so we need to figure out how this circuit solves out with that resistor RPU being present!

We already know what the V-I curve of a MOSFET looks like if we have complete control of its $v_{DS}$ on the x axis:

How does the resistor RPU affect things? Treat the MOSFET as a unknown for a moment like shown below and think about what will $v_{DS}$ be based solely on $i_D$ and the resistor

$R_{pu}$ is a resister... so $i_D \rightarrow V_{Rpu}$

$V_{Rpu}$ follows Ohms Law

$V_{Rpu}$ slope $= R_{pu}$
Then let's flip the axes... Ohm's Law is nothing more than a relationship... doesn't necessarily imply causality.

The plot to the right is a graphical demonstration of the relationship between $v_{DS}$ and $i_D$ that the MOSFET requires.

Each one of these plots was created by "ignoring" the other component. We need to find what will happen when they are put together!

We can do that by placing the graphs on top of one another. In the locations where they overlap, this is where all constraints are satisfied!!! Pretty cool (next page):
We can see that for mid-range $V_{GS}$ values, the system will "land" in the saturation region of the MOSFET! This is where we'll usually try to Bias the circuit for analog amplification purposes. To do this reliably we'll need small signal analysis which will be covered after break:
For extreme values of $v_{GS}$ (in the example of this circuit we'd say less than $V_T$ for "LOW" and ~$V_{DD}$ for "HIGH") the circuit works in two distinct regions:

- **CUTOFF** when $v_{GS}$ is LOW
- **Triode** when $v_{GS}$ is $V_{DD}$

We previously saw that the MOSFET acts like two different equivalent circuits in these two modes. When in:

- **CUTOFF**: it acts like an open (or an infinite ohm resistor)
- **TRIODE**: it acts like a low ohm resistor (value $R_{on}$)

So for cases where $v_{GS}$ ($v_{IN}$) is only High-Low voltages the circuit looks like the following (THE MOSFET ACTS LIKE A VOLTAGE CONTROLLED SWITCH!!!!!!)
THE MOSFET in this setup is acting like a switch having an infinite resistance when "OFF" and a low resistance (Ron) when "ON". In the circuit we just analyzed, this results in the following output:

\[
\text{If } R_{pu} \gg R_{on} \quad \text{(and it usually will be)}
\]

\[
\begin{align*}
\text{when } \, V_{in} = 5 \, \text{V:} \\
V_{out} &= 5 \, \text{V} \left( \frac{R_{on}}{R_{on} + R_{pu}} \right) \approx 5 \, \text{V} \left( \frac{0}{0 + R_{pu}} \right) \approx 0 \, \text{V} \\
\text{when } \, V_{in} = 0 \, \text{V:} \\
V_{out} &= 5 \, \text{V} \left( \frac{R_{off}}{R_{off} + R_{pu}} \right) \approx 5 \, \text{V} \left( \frac{\infty}{\infty + R_{pu}} \right) \approx 5 \, \text{V}
\end{align*}
\]

Hmm mmmm.....

This circuit seems to "opposite" whatever voltage is given to it. If we package up the idea of VDD and 0V into a binary 1 and 0, what we have here is a logical NOT gate!

<table>
<thead>
<tr>
<th>(V_{in})</th>
<th>(V_{out})</th>
</tr>
</thead>
<tbody>
<tr>
<td>VDD</td>
<td>0V</td>
</tr>
<tr>
<td>0V</td>
<td>VDD</td>
</tr>
</tbody>
</table>

\[\text{NOT Gate:}\]

\[
\begin{array}{c|c}
\text{IN} & \text{OUT} \\
0 & 1 \\
1 & 0
\end{array}
\]

We can make more complicated things out of this!! Let's try the following:

What will \(V_{out}\) be for the four possible HIGH/LOW input combinations?

- \(V_{A} = 0V, \, V_{B} = 0V\)
- \(V_{A} = VDD, \, V_{B} = 0V\)
- \(V_{B} = 0V, \, V_{B} = VDD\)
- \(V_{A} = VDD, \, V_{B} = VDD\)
When $V_A = V_{DD}$, $V_B = 0V$

When $V_A = 0V$, $V_B = V_{DD}$

When $V_A = V_{DD}$, $V_B = V_{DD}$

Taken all together this forms a NOR Gate!

<table>
<thead>
<tr>
<th>Inputs</th>
<th>Output</th>
</tr>
</thead>
<tbody>
<tr>
<td>$V_A$</td>
<td>$V_B$</td>
</tr>
<tr>
<td>$0$</td>
<td>$0$</td>
</tr>
<tr>
<td>$V_{DD}$</td>
<td>$0$</td>
</tr>
<tr>
<td>$0$</td>
<td>$V_{DD}$</td>
</tr>
<tr>
<td>$V_{DD}$</td>
<td>$V_{DD}$</td>
</tr>
</tbody>
</table>

Package up as a

\[ A \rightarrow \neg \quad \neg \rightarrow V_{out} \]
NAND and NOR gates are actually sufficient to build anything we want in a computer, believe it or not. We can combine our circuits above to get other logical operations.
Do previous circuit out

<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
<th>Out</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>0</td>
<td>VDD</td>
<td>VDD</td>
</tr>
<tr>
<td>VDD</td>
<td>0</td>
<td>VDD</td>
</tr>
<tr>
<td>VDD</td>
<td>VDD</td>
<td>VDD</td>
</tr>
</tbody>
</table>

This is an OR Gate

We essentially did this

\[ \text{NOR} \quad \text{NOT} \quad = \text{OR} \]

(not not or)

Try this one: (for fun... not required)

Should get:

\[
\begin{array}{c|c|c}
A & B & \text{OUT} \\
0 & 0 & VDD \\
0 & VDD & 0 \\
VDD & 0 & 0 \\
VDD & VDD & VDD \\
\end{array}
\]

XNOR Gate

We used this for multiplying in our lab a few weeks back!!
So several times in lecture Prof Lang or Voldman have described circuits where there are these switches that open and close as needed with specific timing or think back to some of those PWM exercises as well.

Those circuits really do exist, except the switches in all of those problems are essentially MOSFETS jumping between their triode (ON mode) and Cutoff (OFF mode). There are no little magical electric gnomes flipping the switches. Instead they are just voltage-controlled switches as described here.

Pretty much all of digital electronics work by jumping between the two modes we focused on at the end here. (in fact you want to try to avoid the saturation mode if you can !)

We'll use saturation mode when doing things like amplifying audio signals and in order to do that we'll need to use some small signal approximations which Prof Lang will cover after the break.