Lecture 20 - Non-linear components, Method of assumed states

April 30, 2019

Contents:
1. Non-linear components
2. Circuits containing non-linear components
3. Graphical analysis
4. Diode models
5. Method of assumed states to solve diode circuits
6. Half- and full-wave rectifier circuits

Reading Assignment:
Agarwal and Lang, Ch. 4 (§§4.1-4.4); Ch. 16 (§16.1-16.3)

Handouts:
Lecture notes

Announcements:
pset 11 (going out next week) will not be collected/graded
Key question for today…

- How do we deal with circuits that contain components with non-linear i-v characteristics such as diodes?
1. Non-linear components: diode

- Diodes follow an exponential law:

\[ i_D = I_S (e^{qv_D/kT} - 1) \]

- \( I_S \) ≡ saturation current (typically in pA – nA range)
- \( q \) ≡ electron charge (1.6x10^{-19} C)
- \( k \) ≡ Boltzmann constant (1.4x10^{-23} J/K)
- \( T \) ≡ temperature (K)
- \( kT/q \) ≡ thermal voltage (26 mV at 300 K)
\[ i_D = I_S \left( e^{\frac{qv_D}{kT}} - 1 \right) \]
2. Circuits containing non-linear components

- Consider simple resistor-diode circuit:

![Resistor-diode circuit diagram]

- Example: fiber optic transmitter driving laser
- Want to know $i_D$ and $v_D$ for different values of $V$, $R$.
- Try node method.
• Node method. One node:

\[ R \]

\[ V \]

\[ i_R \]

\[ i_D \]

\[ v_D \]

• Node equation:

\[
\frac{v_D - V}{R} + I_S(e^{qv_D/kT} - 1) = 0
\]

• Transcendental equation: no closed-form solution.

• Need better techniques:
  - Numerical analysis
  - Graphical analysis
  - Approximations
3. Graphical analysis

• Not very accurate, but provides useful physical intuition.

• Consider same example again:

![Diode Circuit Diagram]

• Strategy:
  – isolate non-linear component and graph i-v characteristics
  – graph i-v characteristics of rest of circuit on the same chart/axis
  – Solution is intersection of both sets of characteristics.
• i-v characteristics of diode:

\[ i_D = I_S \left( e^{\frac{q v_D}{kT}} - 1 \right) \]

• i-v characteristics of rest of circuit (use same variables: \( i_D \) and \( v_D \)):

\[ i_D = \frac{V - v_D}{R} \]

• Graphical representation:
Graphical technique provides good physical intuition:

- Effect of changing resistance:

- Effect of changing voltage:
4. Approximations: Diode models

- Not all the problems need the same level of accuracy/complexity in the way we represent a diode… Different models.

- Exponential model (model 1):

\[ i_D = I_S \left( e^{\frac{qv_D}{kT}} - 1 \right) \]
• Ideal diode model (model 2):

\[ i_D = \begin{cases} 
0 & \text{for } v_D < 0 \\
> 0 & \text{for } v_D = 0 
\end{cases} \]

**Piecewise linear model:** device described by different linear branches that apply in different regimes

Model is quite crude: diode never dissipates power \( \rightarrow \) problem in power electronics circuits
• A more accurate diode model (model 3):

\[
\text{for } v_D < v_\gamma : i_D = 0 \\
\text{for } v_D = v_\gamma : i_D > 0
\]

Typical values of \(v_\gamma \sim 0.5-0.7\) V (although it depends on the specific diode)

This model now accounts for power dissipation in diode
• An even more accurate diode model (model 4):

\[ i_D = \begin{cases} 
0 & \text{for } v_D < v_\gamma \\
\frac{v_D - v_\gamma}{R} & \text{for } v_D > v_\gamma
\end{cases} \]
• Models typically have trade-offs between accuracy and complexity:

• Think of models as tools in a toolbox:
  → choose the most suitable model for the job at hand

• Simpler models are “piece-wise”:
  – different regions of the i-v characteristics described by different equations
  – discontinuous derivatives at boundaries

• How do we work with piece-wise models?
5. **Method of assumed states to solve diode circuits**

1. Draw equivalent circuits for each of the assumed states of the diode.

2. For each subcircuit:
   - Solve for output
   - Identify range of inputs for which subcircuit applies

3. Assemble complete solution by “piece-wise” splicing partial solutions.
• Example:

\[ i_D \]

With

\[ v_{IN}(t) \]

\[ V_{in} \]

\[ 0 \]

\[ t \]

• Use model 3 for diode:

\[ v_{IN} \]

\[ v_\gamma \]

\[ i_D \]

\[ v_{OUT} \]

for \( v_D < v_\gamma \): \( i_D = 0 \)

for \( v_D = v_\gamma \): \( i_D > 0 \)
1. Draw equivalent circuits for each of the assumed states of the diode.

Diode has two states:
- open (reverse bias) or OFF
- short (forward bias) or ON

\[ \text{for } v_D < v_\gamma : i_D = 0 \]
\[ \text{for } v_D = v_\gamma : i_D > 0 \]
2. For each subcircuit:
   - Solve for output
   - Identify range of inputs for which it applies

   • Diode in short state:

   \[ v_{OUT} = v_{IN} - v_\gamma \]

   \[ i_D = \frac{v_{IN} - v_\gamma}{R} \]

   For short state to apply, \( i_D \geq 0 \) (forward bias):

   \[ v_{IN} \geq v_\gamma \]
• Diode in open state:

\[ v_{OUT} = 0 \]

Voltage across ideal diode:

\[ v_{Di} = v_{IN} - v_\gamma \]

For open state to apply, \( v_{Di} < 0 \) (reverse bias):

\[ v_{IN} \leq v_\gamma \]
3. Assemble complete solution by “piecewise” splicing partial solutions.

\[
\text{for } v_{IN} \leq v_\gamma : \quad v_{OUT} = 0 \\
\text{for } v_{IN} \geq v_\gamma : \quad v_{OUT} = v_{IN} - v_\gamma
\]
6. Full-wave Rectifier

![Diagram of a full-wave rectifier with a sinusoidal signal, half-wave rectification, and full-wave rectification graphs.]

- Input
- DC Output
- Demo
Summary

• Circuits with non-linear devices often result in transcendental equations.
• Graphical technique: approximate method that yields valuable physical intuition.
• A given device can be described by different models.
• Models typically have trade-off between accuracy and complexity: important to select the most suitable model for each situation.
• Method of assumed states: technique to solve circuits involving non-linear elements described by piece-wise model.
How can we use diodes to send 2 bits of information through only one wire?