Lecture 21 - Non-linear components, small-signal analysis

May 2\textsuperscript{nd}, 2019

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1. Incremental analysis

Reading Assignment:
Agarwal and Lang, Ch. 4, § § 4.5, 4.6

Handouts:
Lecture 21 notes

Announcements:
High-Speed Stock Traders Turn to Laser Beams
Anova to Use Laser Devices for Fast Communication of Market Data

By SCOTT PATTERSON
Feb. 11, 2014 11:00 p.m. ET

CEO Michael Persico of Anova Technologies, which is linking stock-market data centers by laser. Claudio Papapietro for The Wall Street Journal

As high-speed stock traders push to trade ever faster, their newest move involves harnessing a technology that U.S. military jets use to communicate as they soar across the sky: lasers.

http://online.wsj.com/news/articles/SB10001424052702303947904579340711424615716
1. Incremental analysis

- Consider an optical transmission system

\[ v_I(t) \rightarrow i_D(t) \rightarrow i_R(t) \rightarrow \text{sound} \]

\[ i_R \propto \text{light intensity in photoreceiver} \]

\[ v_I(t) \]

Data, music, ...

\[ v_I(t) \]

light intensity in photoreceiver

\[ i_D \]

\[ i_R \]

AMP
• Problem: LED is non linear \( \rightarrow \) distortion!

\[
v_I(t) \quad + \quad + \quad v_D \quad - \quad i_D(t)
\]

\[
v_D(t) \quad i_D(t)
\]

\[
v_E
\]

\[
\frac{d}{dt}v_D(t) = i_D(t)
\]
• What we really want is linear characteristics for the LED!
How can we use a non-linear device as a linear one?
• Insight: small region in i-v characteristics looks linear!

• Apply signal on top of DC offset or DC bias:

\[ v_i(t) = V_i + v_i(t) \]
\[ v_D(t) = V_D + v_d(t) \]
• If $v_d(t)$ is small enough, $i_d(t)$ can be close replica of $v_d(t)$
Consider four possible situations:

- **High bias, small signal**
- **High bias, large signal**
- **Low bias, small signal**
- **Low bias, large signal**
Incremental analysis or small-signal method

- Interested in situations where signal has small magnitude:
  - want to know $i_d(t)$ in response to $v_d(t)$

- Key insight: if $v_d(t)$ is small enough, device behavior is roughly linear
  - linearize device $i$-$v$ characteristics
• Relationship between \( v_d(t) \) and \( i_d(t) \):

\[
i_d(t) = \frac{1}{r_d(t)} v_d(t)
\]

With \( r_d \equiv \text{dynamic resistance} \):

\[
r_d(V_D) = \frac{1}{\frac{df(v_D)}{dv_D} \bigg|_{v_D=V_D}}
\]
2. The mathematical description: do Taylor series expansion and select linear term

- In general terms, substitute:

\[ i_D = f(v_D) = f(V_D + v_d) = I_S(e^{qv_D/kT} - 1) = I_D + i_d \]

- First three terms of Taylor series expansion:

\[ i_D = f(V_D) + \frac{df(v_D)}{dv_D} \bigg|_{v_D=V_D} \cdot v_d + \frac{1}{2} \frac{d^2f(v_D)}{dv_D^2} \bigg|_{v_D=V_D} v_d^2 + \cdots \]
• Then:

\[ i_D = I_D + i_d \]

\[ i_D \approx f(V_D) + \frac{df(v_D)}{dv_D} \bigg|_{v_D=V_D} \cdot v_d \]

• Identify terms:
  – Bias (or operating point):

  \[ I_D = f(V_D) \]

  – Small signal:

  \[ i_d(t) = \frac{df(v_D)}{dv_D} \bigg|_{v_D=V_D} \cdot v_d(t) \]

  linear relation between \( v_d(t) \) and \( i_d(t) \).

• Proportionality constant between \( v_d(t) \) and \( i_d(t) \) has units of inverse resistance.

\[ r_d(V_D) = \frac{1}{\frac{df(v_D)}{dv_D} \bigg|_{v_D=V_D}} \]
3. The circuit view: Small-signal technique effectively breaks problem into two simpler problems:
3. The circuit view: Small-signal technique effectively breaks problem into two simpler problems:

\[ v_i(t) = V_i + v_i(t) \]
\[ v_D(t) = V_D + v_d(t) \]

- **Bias circuit:**
  Non-linear (but nothing is changing with time…)

- **Small-signal or incremental circuit:**
  Linear… we can use Superposition, homogeneity, Thevenin and Norton
Example

Consider diode with $I_S=1 \, \text{pA}$ biased at $V_D=0.6 \, \text{V}$ at room temperature. A signal with $1 \, \text{mV}$ amplitude is applied. What is the amplitude of the small-signal current?
Example

Consider diode with $I_S=1$ pA biased at $V_D=0.6$ V at room temperature. A signal with 1 mV amplitude is applied. What is the amplitude of the small-signal current?

Remember i-v characteristics of diode:

$$i_D = I_S(e^{qV_D/kT} - 1)$$

Dynamic resistance of diode:

$$r_d = \frac{1}{df(v_D)/dv_D} = \frac{1}{I_S \frac{q}{kT} e^{qV_D/kT}}$$

This can be simplified if we remember that, at bias point:

$$I_D = I_S(e^{qV_D/kT} - 1)$$

Then:

$$r_d = \frac{kT}{q(I_D + I_S)}$$
Compute $I_D$:

$$I_D \approx 10^{-12} e^{0.6/0.026} = 11 \, mA$$

Dynamic resistance:

$$r_d = \frac{kT}{q(I_D + I_S)} \approx \frac{kT}{qI_D} = \frac{0.026}{0.011} = 2.4 \, \Omega$$

Small-signal current amplitude:

$$i_d = \frac{v_d}{r_d} = \frac{0.001}{2.4} = 0.42 \, mA$$
• How “small” does the small signal need to be?

Analysis good if in Taylor series expansion:

\[
\frac{1}{2} \frac{d^2 f(v_D)}{dv_D^2} \bigg|_{v_D = v_D} v_d^2 \ll \frac{df(v_D)}{dv_D} \bigg|_{v_D = v_D} \cdot v_d
\]

\[
\frac{1}{2} I_S \left( \frac{q}{kT} \right)^2 e^{q v_D/kT} \cdot v_d^2 \ll I_S \left( \frac{q}{kT} \right) e^{q v_D/kT} \cdot v_d
\]

Or:

\[
v_d \ll 2 \frac{kT}{q}
\]

At room temperature, this implies:

\[
v_d \ll 2 \times 0.026 = 0.052 \ V
\]
Summary

- Incremental or small-signal analysis: powerful technique suitable for situations with relatively “small” signals.
- Incremental or small-signal model: fully linearized model suitable to solve the small signal problem; can use many circuit solving techniques.
- Values of elements in small-signal model generally depend on the bias point.