Amplifier Small-Signal Analysis

• Biasing

• Small-Signal Analysis
  – Expansion
  – Taylor Series
  – Small-Signal Models
Where is the amplifier?

Is this an amplifier?

The amplifier behavior is more apparent from this viewpoint.

Look here!
Biaseding & Small Signals

Biaseding and small-signals are used to avoid distortion by nonlinearities. (What if a large signal is desired?)

\[ V_{IN} = V_{IN} + V_{in} \]

\[ V_{OUT} = V_{OUT} + V_{out} \]
MOSFET Amplifier Review

\[ i_D = \frac{K}{2} (V_{GS} - V_t)^2 \]

MOSFET (Saturation): \[ i_D = \frac{K}{2} (V_{GS} - V_t)^2 \]

Amplifier: \[ V_{OUT} = V_s - R \frac{K}{2} (V_{IN} - V_t)^2 \]

\[ \Rightarrow i_D \]
Small-Signal Analysis (Expansion)

Bias: \( V_{\text{out}} = V_S - \frac{RK}{2} (V_{\text{in}} - V_T)^2 \)

Off-Bias: \( V_{\text{out}} = V_{\text{out}} + V_{\text{out}} \)

\[ = V_S - \frac{RK}{2} (V_{\text{in}} - V_T)^2 \]

\[ = V_S - \frac{RK}{2} (V_{\text{in}} + V_{\text{in}} - V_T)^2 \]

\[ = V_S - \frac{RK}{2} (V_{\text{in}} - V_T)^2 \]  \hspace{1cm} \text{Bias}

\[ - RK (V_{\text{in}} - V_T) V_{\text{in}} \]  \hspace{1cm} \text{Linear}

\[ - \frac{RK}{2} V_{\text{in}}^2 \]  \hspace{1cm} \text{Quadratic}

Linearized Small Signals: \( V_{\text{out}} = -RK (V_{\text{in}} - V_T) \frac{V_{\text{in}}}{\text{Small-Signal V Gain}} \)

Accurate if \( V_{\text{in}} - V_T \gg \frac{V_{\text{in}}}{2} \).
Small-Signal Analysis (Differentiation)

\[ V_{out} = V_S - \frac{RK}{2} (V_{IN} - V_T)^2 \]

\[ V_{out} + V_{out} \approx V_{out} \bigg|_{V_{IN}} + \left. \frac{dV_{out}}{dV_{IN}} \right|_{V_{IN}} \cdot V_{IN} \]

\[ = V_{out} - RK (V_{IN} - V_T) V_{IN} \]

\[ V_{out} \approx -RK (V_{IN} - V_T) V_{IN} \]

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Small-Signal Voltage Gain
Numerical Example

1. SS Gain = \(-RK(V_{\text{IN}} - V_T)\)

2. \(I_D = \frac{K}{2}(V_{\text{IN}} - V_T)^2 \Rightarrow V_{\text{IN}} - V_T = \sqrt{\frac{2I_D}{K}}\)

\(\Rightarrow\) SS Gain = \(-R\sqrt{2KI_D}\)

3. Bias point with \(V_{\text{OUT}} = \frac{V_s}{2}\) \(\Rightarrow\)

\(RI_D = \frac{V_s}{2} \Rightarrow I_D = \frac{V_s}{2R}\) \(\Rightarrow\)

SS Gain = \(-\sqrt{RKV_s}\)

4. \(V = 5V\)

\(K \approx 0.14\ A/V^2\) \(\Rightarrow\) SS Gain \(\approx -26\)

\(R = 1000\ \Omega\)
Two-Stage Amplifier Example

Assuming saturation regions of operation:

\[ V_{\text{MID}} = V_s - \frac{R_1 K_i}{2} (V_{\text{IN}} - V_{T1})^2 \]

\[ V_{\text{OUT}} = V_s - \frac{R K_2}{2} (V_{\text{MID}} - V_{T2})^2 \]

\[ V'_{\text{OUT}} = V_s - \frac{R K_2}{2} (V_s - \frac{R_1 K_i}{2} (V_{\text{IN}} - V_{T1})^2 - V_{T2})^2 \]

Banding → \( V_{\text{IN}} \) → \( V_{\text{MID}} \) → \( V_{\text{OUT}} \)

Linearization yields small-signal gain:

\[
\left. \frac{d V'_{\text{OUT}}}{d V_{\text{IN}}} \right|_{V_{\text{IN}}} = \left( -\frac{R_2 K_2}{2} \right) \frac{2 (V_s - \frac{R_1 K_i}{2} (V_{\text{IN}} - V_{T1})^2 - V_{T2}) (\frac{R K_2}{2}) \left( V_s - \frac{R_1 K_i}{2} (V_{\text{IN}} - V_{T1})^2 - V_{T2} \right)}{V_{\text{MID}}} = \frac{R_2 K_2 (V_{\text{MID}} - V_{T2})}{R_1 K_i (V_{\text{IN}} - V_{T1})} \]

Second-stage gain

First-stage gain

No inter-stage loading because \( i_G = 0 \).
The Problem

- Small-signal analysis yields valuable insight. But ...

- Small-signal analysis via expansion or differentiation requires a complete closed-form nonlinear analysis.

- The expansion or differentiation can be very complex for a large circuit.

- Numerical analysis is possible but it yields little insight.
The Solution

- Rather than analyze first and then linearize the analysis, linearize first and then analyze.

- Remove the biases, from the linearization, to uncover the small-signal model.

- The linearization still requires an operating point, but it can come from a non-linear analysis, a numerical analysis, an approximate analysis, or an experiment.
Small-Signal Modeling Summary

Four steps to develop a small-signal model.

• Carry out a (nonlinear) large-signal bias analysis (analytical, graphical, numerical, experimental). The small-signal model operates around this bias, showing variations from this bias.

• Linearized KVL and KCL set the small-signal model circuit topology. Since KVL and KCL are linear, the small- and large-signal models have the same topology.

• Separate large-signal inputs (independent sources) into bias inputs plus small-signal inputs, and remove/zero the biases. (A bias voltage becomes a short circuit, and a bias current becomes an open circuit.)

• Replace all other devices with their linearization around their bias points, removing the bias in the process. The resulting circuit is the small-signal model.
Treatment Of Independent Sources

\[ V_{IN} = V_{in} \]
\[ i_{IN} = I_{IN} \]

If the independent source has no small-signal component, then...

\[ V_{IN} \]
\[ i_{IN} \]
Linearizing A Dependent Source

Bias: \( I_{DS} = f(V_c) \)

Off Bias: \( i_{DS} = f(V_c) \)

\[ = I_{DS} + i_{ds} \]

\[ = f(V_c + V_c) \]

\[ = f(V_c) + \frac{df}{dV_c} \bigg|_{V_c} \cdot V_c \]

\[ = I_{DS} + g_{ds} \cdot V_c \]

\[ = I_{DS} + g_{ds} (V_c - V_G) \]
MOSFET Small-Signal Model

Saturation Region

\[ i_G = 0 \]
\[ V_{GS} \]
\[ V_{DS} \]
\[ \frac{K}{2} (V_{GS} - V_T)^2 \]

Small-Signal Model

\[ i_d \]
\[ V_{GS} \]
\[ V_{DS} \]
\[ g_m V_{GS} \]

\[ g_m = K (V_{GS} - V_T) \]
Two-Stage Amplifier Example

\[ V_{in} \]  \[ \rightarrow \]  \[ V_{gs1} \]

\[ V_{gs1} \]  \[ \rightarrow \]  \[ V_{gs2} \]

\[ V_{gs2} = -R_1 K_1 (V_{in} - V_{T1}) V_{in} \]
\[ V_{out} = -R_2 K_2 (V_{M10} - V_{T2}) V_{gs2} \]
\[ V_{Out} = R_1 K_1 (V_{in} - V_{T1}) R_2 K_2 (V_{M10} - V_{T2}) V_{in} \]