Coupled Resonators

- Transformers
- Wireless Power Xfer
- Tesla Coil
- Coupled Resonators
Two Poorly-Coupled Transformers

Wireless Power Transfer

Tesla Coil
Transformer Physics

\[ \mathbf{H} \cdot \mathbf{l} = N_1 \mathbf{i}_1 + N_2 \mathbf{i}_2 \Rightarrow \mathbf{H} = \frac{(N_1 \mathbf{i}_1 + N_2 \mathbf{i}_2)}{l} \]

Gauss: \[ \phi = \mu \mathbf{H} \cdot \mathbf{A} = L_0 (N_1 \mathbf{i}_1 + N_2 \mathbf{i}_2) \]

\[ L_0 = \frac{\mu \mathbf{A}}{l} \]

\[ \lambda_1 = N_1 \phi = L_0 (N_1^2 \mathbf{i}_1 + N_1 N_2 \mathbf{i}_2) \]

\[ \Rightarrow \lambda_2 = N_2 \phi = L_0 (N_1 N_2 \mathbf{i}_1 + N_2^2 \mathbf{i}_2) \]

Leakage: \[ \lambda_1 = N_1 \phi + L_{11} \mathbf{i}_1 \text{ and } \lambda_2 = N_2 \phi + L_{12} \mathbf{i}_2 \]

\[ \begin{bmatrix} \lambda_1 \\ \lambda_2 \end{bmatrix} = \begin{bmatrix} N_1 L_0 + L_{11} & N_1 N_2 L_0 \\ N_1 N_2 L_0 & N_2 L_0 + L_{12} \end{bmatrix} \begin{bmatrix} \mathbf{i}_1 \\ \mathbf{i}_2 \end{bmatrix} = \begin{bmatrix} L_1 & M \\ M & L_2 \end{bmatrix} \begin{bmatrix} \mathbf{i}_1 \\ \mathbf{i}_2 \end{bmatrix} \]

\[ L_{12} > M^2 \]

Faraday: \[ \mathbf{v}_1 = \frac{d \lambda_1}{dt} \text{ and } \mathbf{v}_2 = \frac{d \lambda_2}{dt} \]
Transformer Circuit Model

\[ V_1 = L_{L1} \frac{di_1}{dt} + N_1^2 L_0 \frac{di_1}{dt} + N_1 N_2 L_0 \frac{di_2}{dt} \]

\[ = L_{L1} \frac{di_1}{dt} + N_1^2 L_0 \left( \frac{di_1}{dt} + \frac{N_2}{N_1} \frac{di_2}{dt} \right) \]

\[ V_2 = L_{L2} \frac{di_2}{dt} + N_2^2 L_0 \frac{di_2}{dt} + N_1 N_2 L_0 \frac{di_1}{dt} \]

\[ = L_{L2} \frac{di_2}{dt} + \frac{N_2}{N_1} \left( N_1^2 L_0 \frac{di_1}{dt} + N_1 N_2 L_0 \frac{di_2}{dt} \right) \]

Ideal Transformer
Wireless Power Transfer

Witricity & Google Images
Wireless Power Transfer Challenge

Assume Matched coils

⇒ $N_1 = N_2$

⇒ $L_{L1} = L_{L2}$

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**Diagram:**

- A diagram of a transformer with labeled parts:
  - Signal Generator
  - Big
  - Small
  - Load
  - Voltage Divider
  - Voltage Divider

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**Graph:**

- A graph showing the Inductance Ratio vs. Center-Center Separation with data points labeled 'Stacked' and 'Sideways'.

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17 cm
A Wireless Power Transfer Solution

Choose \( L - C_s \) resonance \( \Rightarrow \omega L = 1/\omega C_s \)

\[
\tilde{V}_{th} = \frac{j\omega M V_A}{R_s + \frac{1}{j\omega C_s} + j\omega L} = \frac{j\omega M}{R_s} V_A
\]

\[
Z_{th} = j\omega (L-M) + \frac{j\omega M (R_s + \frac{1}{j\omega C_s} + j\omega L - j\omega M)}{R_s + \frac{1}{j\omega C_s} + j\omega L} = \frac{\omega^2 M^2}{R_s} + j\omega L
\]

Choose \( L - C_L \) resonance \( \Rightarrow \omega L = 1/\omega C_L \), and
\( R_L = \frac{\omega^2 M^2}{R_s} \) to achieve maximum power transfer.

Double resonators!

\[
R_L = \frac{\omega^2 M^2}{R_s}
\]

Power \( = \frac{V_A^2}{2R_s} \)
Matching Network

What if \( R_L \neq \frac{\omega^3 M^2}{R_S} \)? Matching network!

\[
Z_M = \frac{R_L}{R_L + \frac{1}{j\omega C_m}} + j\omega L_M = \frac{R_L - j\omega R_L^2 C_m}{1 + \omega^2 R_L^2 C_m^2} + j\omega L_M
\]

\[
= \frac{R_L}{1 + \omega^2 R_L^2 C_m^2} - \frac{j\omega R_L^2 C_m}{1 + \omega^2 R_L^2 C_m^2}
\]

Choose \( C_m \) to reduce \( R_L \), and \( L_M \) to obtain zero reactance.

For a substantial reduction of \( R_L \),

\[
\omega^2 R_L^2 C_m^2 \ll 1 \Rightarrow Z_M \approx \frac{R_L}{\omega^2 R_L^2 C_m^2} + \frac{1}{j\omega C_M} + j\omega L_M
\]
Demo

For an LED load, voltage is more important than load matching, so choose a resonator output over a load matching output.

\[
\begin{align*}
V_s &= 10 \ V_{pk} \\
L &= 230 \ \mu H \\
C &= 9.6 \ \text{nF} \\
\omega &= 2\pi \times 107.1 \ \text{krad/s} \\
M &\sim \frac{L}{10} \ \text{to} \ \frac{L}{200} \\
R_L &\sim 1 \ \text{k}\Omega \ \text{to} \ 5 \ \text{k}\Omega \\
R_s &= 50 \ \Omega \\
R &= 0.5 \ \Omega \\
\omega &= \frac{1}{\sqrt{L/C}} \\
\text{Primary } Q &= \frac{\sqrt{L/C}}{R_s} = 3 \implies V_P = 30 V_{pk} \\
\text{For large separation, } Z &\approx R + j\omega \\
\text{For large separation, secondary } Q &\approx \frac{R_L}{\sqrt{L/C}} \approx 20 \\
\tilde{V}_{th} &= \frac{\omega M}{R_s} V_A \approx \frac{M}{L} 30 \tilde{V}_{pk}
\end{align*}
\]
Tesla Coil - Resonant Transformer

Purpose: high voltages at high frequency

- Air core for low core loss, high breakdown voltage and high resonance frequency
- Single layer of thin tightly packed turns to balance high turns ratio, high breakdown voltage and low eddy-current loss
- Short primary and long secondary for high turns ratio
- Poor primary-secondary coupling
The Tesla Coil System employs two coupled resonators with both resonators tuned to the same frequency.
What comes after 6.002?

- Labs:
  - 6.101 analog circuits
  - 6.111 digital circuits
  - 6.115 embedded systems
  - 6.131 power electronics
- 6.012 semiconductor devices to computing systems
  - 6.301 solid state circuits
    - 6.775 CMOS circuits
    - 6.776 high-speed circuits
  - 6.334 power electronics
- 6.374 digital integrated circuits
- 6.720 integrated microelectronic devices
- 6.690 Electric power systems