Outline
- Capacitors
- Step response

(1) Capacitors.
There are 4 fundamental passive components:
- Resistor R
- Capacitor C
- Inductor L
- Transformer

Today we will focus on the C.

Why capacitors?
- Basic building block for transistors
- Useful for manipulating signals - filters, memory
- Widely used in transducers
- Key in (modelling) energy storage

Positive and negative charges attract each other?
What happens when we prevent them from touching each other → Charge separation → energy storage in the form of electrical potential → Capacitor
\[ C = \frac{\varepsilon A}{d} \]
\[ q = C \cdot V_c \quad \text{(linear relationship)} \]
\[ i = \frac{dq}{dt} = C \frac{dV_c}{dt} = \frac{C}{d} \frac{dV_c}{dt} = i \]
\[ \text{Device law} \]
\[ C \text{ of } f(t) \]

\[ V_c(t) = \frac{1}{C} \int_{0}^{t} i(t) \, dt = \frac{1}{C} \left( \int_{0}^{t} i(t) \, dt + \int_{-\infty}^{0} i(t) \, dt \right) \]

\[ \text{Current} \]

\[ \frac{V_c(t)}{C} = \frac{V(t)}{C} \]

\[ V(t) \quad \text{(State)} \]

\[ \text{Steady energy?} \]
\[ W_e(t) = \int_{-\infty}^{t} P \, dt = \int_{-\infty}^{t} V(t) \cdot i(t) \, dt = \int_{-\infty}^{t} V_c(t) \cdot C \frac{dV_c(t)}{dt} \, dt \]
\[ = \int_{0}^{V} C \cdot V_c(t) \cdot dV_c = \frac{1}{2} CV^2 = W_e(V) \]
Series & Parallel:

\[ C_1 + C_2 = \frac{1}{C_T} = \frac{1}{C_1} + \frac{1}{C_2} \]

**Step Response**

Two Approaches:

1. **Intuition**
2. **Math:**

KVL:

\[ V_S - i \cdot R = V_c \]

\[ V_S = V_c + R \cdot C \cdot \frac{dV_c}{dt} \]

First order LDE
\[ V_{en} \rightarrow \text{Set } v_{ch} \text{ to } 0. \]

\[ V_{en} + RC \frac{dv_{en}}{dt} = 0 \]

\[ v_{en} = Ae^{st} \]

\[ Ae^{st} + RCsAe^{st} = 0 \]

\[ 1 + RCs = 0 \rightarrow s = \frac{-1}{RC} \]

\[ v_{en} = Ae^{-t/RC} \]

\[ Z = RC \text{ (time constant)} \]

\[ v_{cp} \rightarrow v_{cp} + RC \frac{dv_{cp}}{dt} = V_o, \quad t > 0 \]

\[ v_{cp} = V_o. \]

\[ v_c = V_o + Ae^{-t/RC}, \quad t > 0. \]

\text{Apply initial condition:}

\[ v_c = 0 \text{ for } t < 0. \]

\[ \text{Unless } i \rightarrow 0, \quad v_c(0^-) = V_c(0^+) \]

\[ 0 = V_o + A \quad \text{at } t = 0^+ \rightarrow A = -V_o \]

\[ v_c(t) = V_o \left(1 - e^{-t/RC}\right), \quad t > 0. \]