6.002 Recitation - Spring 2019
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Outline
Second Order Op-Amp Filters, Sallen-Key Filter

Review of Op-amps:

Examples of First order Filters

Low Pass Filter

Now consider an op-amp circuit:

\[
\frac{V_{in}}{R_1} + \left( \frac{1}{R_2} + j\omega C \right) V_o = 0
\]

\[
\frac{V_{in}}{R_1} = -\left( \frac{1}{R_2} + j\omega C \right)V_o
\]

\[
\frac{V_{in}}{V_o} = \frac{-1}{R_1(1 + j\omega RC)} = \frac{-1}{R_1 \left( \frac{R_2}{R_1} + j\omega R_2 C \right)}
\]

This acts like a low pass filter and gives amplification with a gain of \( \frac{R_2}{R_1} \).
High Pass Filter

\[
\frac{1}{\jmath \omega C}
\]

\( V_{in} \)

\[ V_{o} = \frac{R}{R_1 + \frac{1}{\jmath \omega C}} V_{in} \]

\[ V_{o} = \frac{j\omega RC}{j\omega RC + 1} V_{in} \]

\[ \frac{V_{o}}{V_{in}} = \frac{j\omega RC}{j\omega RC + 1} \]

Now consider an op-amp circuit:

\[ \frac{V_{in}}{R_1 + \frac{1}{\jmath \omega C}} + \frac{V_{o}}{R_2} = 0 \]

\[ \frac{V_{in}}{R_1 + \frac{1}{\jmath \omega C}} = -\frac{V_{o}}{R_2} \]

\[ V_{o} = -\frac{R_2}{R_1 + \frac{1}{\jmath \omega C}} = -\frac{R_2}{R_1} \frac{j\omega RC}{1 + j\omega CR_1} \]

This acts as a high-pass filter with a gain of \( \frac{R_2}{R_1} \)
Let's consider the following design:

\[(V_o - V_{in})(\frac{1}{R_1} + j\omega C) = \frac{V_{in}}{R_2}\]

\[V_o - V_{in} = \frac{V_{in}}{R_2(\frac{1}{R_1} + j\omega C)}\]

\[V_o = V_{in}(1 + \frac{1}{R_2(\frac{1}{R_1} + j\omega C)})\]

\[\frac{V_o}{V_{in}} = 1 + \frac{R_2}{R_1} \frac{1}{1 + j\omega R_1 C}\]

**Gain when \(\omega = 0\), \(\frac{V_o}{V_{in}} = 1 + \frac{R_1}{R_2}\)** and if \(R_2 \gg R_1\), \(\frac{V_o}{V_{in}} \approx \frac{R_1}{R_2}\)

**Cut-off frequency** \(\omega_c = \frac{1}{R_1 C}\)

Let's look at the first-order passive low-pass filter again:

\[\frac{V_o}{V_{in}} = -\frac{R_2}{R_1} \frac{1}{1 + j\omega R_2 C}\]

**Gain** = \(\frac{R_2}{R_1}\)

**\(\omega_c = \frac{1}{R_2 C}\)**

To make the slope steeper and filter performance more ideal, we can use higher-order filters.
Second Order Active Filters

One way to achieve higher order filters is to use cascading two first order filters.

\[ \frac{V_o}{V_{in}} = \frac{V_o}{V_a} \cdot \frac{V_a}{V_{in}} \]

\[ \frac{V_o}{V_{in}} = \left( \frac{C_2}{R_1} \right)^2 \frac{1}{1+j2\omega R_2 C_2 - \omega^2 R_2^2 C_2} \]

For \( \omega \gg \frac{1}{R_2 C} \) \( |\frac{V_o}{V_{in}}| \propto \frac{1}{\omega^2} \) The slope increases.

Alternatively, a second order filter can also be realized by using just 1 op-amp.

\[ \frac{V_{in} - V_a}{R_1} + (V_o - V_a) j \omega C_1 + \frac{V_o - V_a}{R_2} = 0 \quad (1) \]

\[ V_o j \omega C_2 + \frac{V_o - V_a}{R_2} = 0 \quad \Rightarrow \quad V_o j \omega C_2 R_2 = V_a - V_o \]

\[ V_a = V_o \left( 1 + j \omega C_2 R_2 \right) \] Substitute this in (1)

\[ \frac{V_o}{V_{in}} = \frac{1}{(j\omega)^2 + j\omega \left( \frac{1}{R_2 C_2} + \frac{1}{R_1 C_1} \right) + \frac{1}{R_1 R_2 C_1 C_2}} \]

For large frequencies \( |\frac{V_o}{V_{in}}| \propto \frac{1}{\omega^2} \)

Cut off frequency \( \omega_c = \frac{1}{\sqrt{R_1 R_2 C_1 C_2}} \)

\[ Q = \frac{\omega_c}{2\alpha} \quad \alpha = \frac{1}{R_1 C_1} + \frac{1}{R_2 C_2} \]
This is similar to an RLC circuit that was covered before but here we don't use an inductor.

\[ H(j\omega) = \frac{(\frac{R}{L})j\omega}{(j\omega)^2 + (\frac{R}{L})j\omega + \frac{1}{LC}} \]

\[ \omega_c = \frac{1}{\sqrt{LC}} \]

\[ \phi = \frac{R}{L} \]

The Sallen-Key filter was invented in 1955 by engineers in Lincoln Lab. As shown here, the Sallen-Key filter design is a second-order filter design, requiring one op-amp for gain control and four passive RC components to achieve tuning.

(Note that by adding \( R_A \) and \( R_B \) you can control the gain)

Example of Second Order High Pass Filter

\[ \text{Gain} = 1 + \frac{R_A}{R_B} \]

\[ \omega_c = \frac{1}{\sqrt{R_A} \cdot \frac{1}{C_1 C_2}} \]