6.002 Recitation - Spring 2019
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Outline
Review of the lecture – Series and Parallel Simplifications, Node Analysis
Demo
Node Analysis Examples

Series and Parallel Simplifications

**Series**

(share the same current)

\[ R_T = R_1 + R_2 + R_3 + \ldots + R_N \]

\[ V_K = \frac{V_T}{R_1 + R_2 + \ldots + R_N} \]

**Parallel**

(share the same voltage)

\[ \frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \ldots + \frac{1}{R_N} \]

\[ G_T = G_1 + G_2 + G_3 + \ldots + G_N \]

\[ i_T = \frac{G_T}{G_1 + G_2 + \ldots + G_N} \]

Voltage Divider

Node Analysis

Five-step process:
1. Assign reference node ("ground") and define its potential to be 0 V
2. Assign node voltages with respect to the ground node
3. Write KCL for each node that has an unknown node voltage
   - Express currents in terms of the node voltage differences and element parameters
4. Solve equations from step 3 for node voltages.
5. Solve for branch voltage and currents.
Nodal Analysis Example 1 – Demo

Step 1: Assign reference node (ground)
Step 2: Assign node voltages
Step 3: Write KCL

Node 1
\[
\begin{align*}
1_i - i_2 - i_3 &= 0 \\
\frac{(V_o - e_1)}{R_1} - \frac{e_1}{R_2} - \frac{(e_1 - e_2)}{R_3} &= 0 \\
(V_o - e_1)g_1 - e_1g_2 - (e_1 - e_2)g_3 &= 0 \\
(g_1 + g_2 + g_3)e_1 - g_3e_2 &= g_1V_o
\end{align*}
\]

Node 2
\[
\begin{align*}
i_3 + i_4 - i_5 + I &= 0 \\
\frac{(e_1 - e_2)}{R_4} + \frac{(V_o - e_2)}{R_5} - \frac{e_2}{R_5} + I &= 0 \\
(e_1 - e_2)g_3 + (V_o - e_2)g_4 - e_2g_5 + I &= 0 \\
g_3e_1 + (g_3 + g_4 + g_5)e_2 &= g_4V_o + I
\end{align*}
\]

\[
\begin{pmatrix}
g_1 + g_3 + g_5 & -g_3 \\
-g_3 & g_4 + g_3 + g_5
\end{pmatrix}
\begin{pmatrix}
e_1 \\
e_2
\end{pmatrix}
=
\begin{pmatrix}
g_1V_o \\
g_4V_o + I
\end{pmatrix}
\]

Step 4: Solve for node voltages (using Cramer's rule)

\[
\begin{pmatrix}
e_1 \\
e_2
\end{pmatrix}
=rac{1}{(g_1 + g_3 + g_5)(g_3 + g_4 + g_5) - g_3^2}
\begin{pmatrix}
g_3 + g_4 + g_5 & g_3 \\
g_3 & g_1 + g_2 + g_3
\end{pmatrix}
\begin{pmatrix}
g_1V_o \\
g_4V_o + I
\end{pmatrix}
\]

solve for \(e_1\) and \(e_2\)

Step 5: Solve for branch voltages and currents

\[
\text{example} \rightarrow \quad V_3 = e_1 - e_2 \quad \text{\(i_3 = \frac{V_3}{R_3} = \frac{e_1 - e_2}{R_3}\)}
\]

NOTE:
\[
\begin{pmatrix}
g_{11} & g_{12} \\
g_{21} & g_{22}
\end{pmatrix}
\begin{pmatrix}
e_1 \\
e_2
\end{pmatrix}
=
\begin{pmatrix}
g_1V_o \\
g_4V_o + I
\end{pmatrix}
\]

\(g_{11}, g_{22}\) (Diagonal Elements) are self conductances (conductances connected to a node)
\(g_{12}, g_{21}\) are mutual conductances (connection between nodes)
Nodal Analysis Example 2

Step 1: Assign the reference node (ground)
Step 2: Assign node voltages
Step 3: Write KCL

Node 1:
\[
\begin{align*}
\bar{i}_1 - \bar{i}_2 - \bar{i}_3 &= 0 \\
\frac{(V_0 - e_1)}{R_1} - \frac{e_1}{R_2} - \frac{(e_1 - e_2)}{R_3} &= 0 \\
(V_0 - e_1)R_1 - e_1R_2 - (e_1 - e_2)R_3 &= 0 \\
(G_1 + G_2 + G_3)e_1 - G_3e_2 &= G_1V_0
\end{align*}
\]

Node 2:
\[
\begin{align*}
\bar{i}_3 - \bar{i}_4 + \bar{I} &= 0 \\
\frac{(e_1 - e_2)}{R_3} - \frac{e_2}{R_4} + \bar{I} &= 0 \\
(e_1 - e_2)G_3 - e_2G_4 + \bar{I} &= 0 \\
-G_3e_1 + (G_3 + G_4)e_2 &= \bar{I}
\end{align*}
\]

\[
\begin{bmatrix}
G_1 + G_2 + G_3 & -G_3 \\
-G_3 & G_3 + G_4
\end{bmatrix}
\begin{bmatrix}
e_1 \\
e_2
\end{bmatrix}
= 
\begin{bmatrix}
G_1V_0 \\
\bar{I}
\end{bmatrix}
\]

Step 4: Solve for node voltages (using Cramer's rule)
\[
e_1 = \frac{V_0(G_1G_3 + G_1G_4) + IG_3}{G_1G_3 + G_1G_4 + G_2G_3 + G_2G_4 + G_3G_4}
\]
\[
e_2 = \frac{G_1G_3V_0 + (G_1 + G_2 + G_3)I}{G_1G_3 + G_1G_4 + G_2G_3 + G_2G_4 + G_3G_4}
\]

Step 5: Solve for branch voltages and currents
Nodal Analysis Example 3

Step 1: Assign the reference node
Step 2: Assign node voltages
Step 3-4: Write KCL and solve

\[ \text{node 1} \]
\[ i_1 - i_2 - I = 0 \]
\[ \frac{(e_3 - e_1)}{R_1} - \frac{e_1}{R_2} - I = 0 \]
\[ \frac{8V - e_1}{100Ω} - \frac{e_1}{100Ω} - 2A = 0 \]
\[ e_1 = -9.6V \]

\[ \text{node 2} \]
\[ i_3 - i_4 + I = 0 \]
\[ \frac{(e_3 - e_2)}{R_3} - \frac{e_2}{R_4} + I = 0 \]
\[ \frac{8V - e_2}{200Ω} - \frac{e_2}{300Ω} + 2A = 0 \]
\[ e_2 = 244.8V \]

Step 5: Solve for branch voltages and currents

\[ V_1 = e_3 - e_1 = 8 - (-9.6) = 104V \]
\[ V_2 = e_1 = -9.6V \]
\[ V_3 = e_3 - e_2 = 8 - 244.8 = -236.8V \]
\[ V_4 = e_2 = 244.8V \]

Double check KCL

\[ \text{node 1} \]
\[ i_1 - i_2 - I = 0 \]
\[ 1.04A - (-0.96A) - 2A = 0 \]

\[ \text{node 2} \]
\[ i_3 - i_4 + I = 0 \]
\[ -1.184A - 0.816A + 2A = 0 \]