6.002 Recitation - Spring 2019
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Outline
Capacitors and RC Circuits
Stability – Equilibria, Comparators, Schmitt Trigger

Capacitors and RC Circuits

Capacitors – Device Law
\[ q = CV_c \]
\[ \frac{d}{dt} q = C \frac{dV_c}{dt} \]
\[ \frac{\psi(t)}{V_c(t)} = \frac{1}{t} \quad \text{(increased total area)} \]
\[ C = C_1 + C_2 \quad \text{(increased total gap distance)} \]

Parallel
\[ C = C_1 + C_2 \]

Series
\[ C = \frac{C_1 C_2}{C_1 + C_2} \]

RC Circuit
\[ V_0(t) = \begin{cases} 0 & t < 0 \\ V_0 & t \geq 0 \end{cases} \]

KVL
\[ V_0(t) = R_i(t) + V_c(t) \]
\[ i_R = \frac{dV_c(t)}{dt} = RC \frac{dV_c(t)}{dt} \]

\[ V_c(t) = V_0 (1 - e^{-t/\tau}) \quad \text{where} \quad \tau = RC \]

Looks like open circuit
Stability – Equilibria, Comparators, Schmitt Trigger

Review:
**Operational Amplifier (Op-Amp)**

*ideal op-amp is a voltage-controlled voltage source: \( V_o = A(V_in-V_r) \)*

* A \( \rightarrow \infty \) (A is the open loop gain)
* \( R_o \rightarrow 0 \)
* \( I_o \rightarrow 0 \) (\( i_o = 0 \) and \( i_e = 0 \))

open-loop op-amp

large gain but unstable

closed-loop op-amp

low gain but stable

**Op-Amp: Example**

1) \( V_o = V_i = V_i \)
2) \( I_1 = \frac{V_i - V_o}{R_1} \)
3) \( I_1 = I_2 = \frac{V_i - V_o}{R_1} \)

4) \( V_o = V_1 + V_2 + I_2 (R_2 + R_3) \)

\( V_o = V_1 + V_2 + \left( \frac{V_i - V_o}{R_1} \right) (R_2 + R_3) \)

\( V_o = V_1 + V_3 + \left( V_1 - V_o \right) \left( \frac{R_2 + R_3}{R_1} \right) \)

In solving op-amp circuits assume:

\( A \rightarrow \infty \)

\( i_o = 0 \)

\( V^+ = V^- \)
Stability

- Equilibria
- Dynamics & Stability
- Comparators
- Positive Feedback
- Schmitt Trigger
- Relaxation Oscillator
- Review Method of Assumed States

**Op-Amp Dynamics**

\[
\begin{align*}
V_{in} &= V_o + V_{out}^M \\
&\Rightarrow V_{out} = A \left( V_{in} + V_{out}^M \right)
\end{align*}
\]

**Equilibria**

\[
\begin{align*}
V_1 &= V_{in} \quad (\text{if } V_{out} = 0) \\
V_2 &= V_{in} \quad (\text{if } V_{out} = 0)
\end{align*}
\]

**Do these two circuits really behave in the same way?**

**Amplifier Dynamics I**

\[
\begin{align*}
V_{out} &= A \cdot V_e \\
V_e &= \frac{R_2}{R_1} \cdot V_{out} \\
&\Rightarrow V_{out} = \frac{A}{R_2} \cdot V_{out} \\
&\Rightarrow V_{out} = \left( \frac{A}{R_2} + 1 \right) V_{out}
\end{align*}
\]

\[
\begin{align*}
\frac{dV_{out}}{dt} &= A \left( V_{in} \right) \frac{V_{out}}{RC} \\
&\Rightarrow V_{out} = 0
\end{align*}
\]
Amplifier Dynamics II

\[ \frac{d}{dt} V_{\text{out}} + \frac{1}{C} V_{\text{out}} = 0 \quad \tau = \frac{RC}{A(\delta - \delta_a)} \]

\[ V_{\text{out}}(t) = V_{\text{out}}(0) e^{-t/\tau} \]

\[ \begin{align*}
\delta &> \delta_a & \text{Negative Feedback} \\
\delta &= \delta_a & \text{Neutral Feedback} \\
\delta &< \delta_a & \text{Positive Feedback}
\end{align*} \]

Op Amp Versus Comparator I

In 6.002, the device

represents an amplifier, usually having ideal properties. It can be used in both negative-feedback amplification (Op-Amp) applications, and positive-feedback comparator (Comparator) applications.

Comparators

- Positive Supply
- Negative Supply

- Fast
- Slow

Op Amp Versus Comparator II

Op-Amp \rightarrow Designed to be largely linear, and for use in negative feedback applications. Implemented with a few high-gain stages having modest bandwidth.

Comparator \rightarrow Designed to be very fast, and for use in open-loop applications or with positive feedback. Often implemented with many very fast stages having modest gain. May employ positive feedback for high-speed.
Schmitt Trigger I

Could interchange roles of \( V_N \) and \( V_{REF} \):

\[ V_N < V_{REF} \quad \text{Fast} \quad V_N > V_{REF} \]

\[ V_{OUT} = -V_s \]

Schmitt Trigger Application

Digital Data \( \rightarrow \) "ST." \( \rightarrow \) Digital Data

Oscillator

\[ V_{OUT} \]

Voltages

\[ V_s \]

\[ \Delta \]

\[ \Delta - \Delta\]

\[ -V_s \]