6.002 Recitation - Spring 2019
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Outline
Inductors
Series and Parallel Connections
RL Circuits

**Inductors**

\[
\text{Flux density } \phi(t) = \frac{N N_i(t)}{A(t)}
\]

\[
\text{Flux linkage } \lambda(t) = N \phi(t) = N A(t) B(t)
\]

\[
\lambda(t) = \frac{\mu N^2 A(t) i(t)}{\ell(t)} \quad \text{where } L(t) = \frac{\mu N^2 A(t)}{\ell(t)}
\]

\[
\lambda(t) = L(t) i(t)
\]

**Device law for time invariant inductor:**

\[
V(t) = \frac{d\lambda(t)}{dt} = \frac{d}{dt} (L_i(t)) = L c i(t)
\]

\[
V(t) = L \frac{di(t)}{dt}
\]

\[
i(t) = \frac{1}{L} \lambda(t) = \frac{1}{L} \int V(t) dt
\]

**Power and Energy**

\[
P(t) = i(t) V(t) = i(t) \frac{d\lambda(t)}{dt} = i(t) L \frac{di(t)}{dt} = L \frac{di(t)}{dt}
\]

\[
W(t) = \int_{t_0}^{t} P(t) dt = \int_{t_0}^{t} L i(t) di(t) = \frac{1}{2} L i(t)^2 \bigg|_{t_0}^{t}
\]

\[
W(t) = \frac{1}{2} L i(t)^2 \quad \text{if } i(t_0) = 0
\]
Parallel and Series Connections

Series Combinations:

\[ V_L = V_{L_1} + V_{L_2} = L_1 \frac{di_1}{dt} + L_2 \frac{di_2}{dt} \]

\[ i_L = i_{L_1} = i_{L_2} \]

Parallel Combinations:

\[ \frac{di_L}{dt} = \frac{V_L}{L} = \frac{V_{L_1}}{L_1} + \frac{V_{L_2}}{L_2} = \left( \frac{1}{L_1} + \frac{1}{L_2} \right) V_L = \frac{V_L}{L} \]

Arguments for continuity of current:

Let \( t - t_0 = \epsilon > 0 \) and let \( \epsilon \to 0 \)

\[ i_L(t) - i_L(t_0) = \int_{t_0}^t \frac{V_L(t')}{L} \, dt' = \frac{V_L \epsilon}{L} \quad \text{Assuming} \ V_L \text{does not change over} \ t \]

As \( \epsilon \to 0 \); \( \frac{V_L \epsilon}{L} \to 0 \) unless \( V_L \to \infty \)

Hence, \( i_L(t_0 + \epsilon) - i_L(t_0) \to 0 \) \( \Rightarrow \) \( i_L(t_0 + \epsilon) = i_L(t_0) \)

\[ i_L(t) \text{ is continuous if} \ V_L \text{ is finite} \]
RL Circuits

Step response:

\[ V_s(t) = V_R + V_L \]
\[ V_L(t) = R i_L(t) + L \frac{d i_L(t)}{dt} \]

Define \( \tau = \frac{L}{R} \)

First order differential equation

Homogeneous solution = \( A e^{-t/\tau} \)

Particular solution = \( \frac{V_I}{R} \)

\[ i_L(t) = A e^{-t/\tau} + \frac{V_I}{R} \]

Use initial condition to find \( A \rightarrow i_L(0)=0 \)

\[ i_L(0) = A + \frac{V_I}{R} = 0 \]

\[ A = -\frac{V_I}{R} \]

\[ i_L(t) = -\frac{V_I}{R} e^{-t/\tau} + \frac{V_I}{R} = \frac{V_I}{R} \left( 1 - e^{-t/\tau} \right) \text{ for } t \geq 0 \]

\[ V_L(t) = L \frac{d i_L(t)}{dt} = V_I e^{-t/\tau} \]

\[ V_R(t) = R i_L(t) = V_I \left( 1 - e^{-t/\tau} \right) \]
What do the waveforms look like?

\[ i_L(t) = \frac{V_I}{R} \left(1 - e^{-\frac{t}{\tau}}\right) \quad \text{for } t \geq 0 \]

\[ V_R(t) = V_I \left(1 - e^{-\frac{t}{\tau}}\right) \]

\[ V_L(t) = V_I e^{-\frac{t}{\tau}} \]

Note:
- Initially (short-time) \( i_L = 0 \) \( \Rightarrow \) \( L \) looks like an open circuit.
- Finally (long-time) \( \frac{di_L}{dt} = 0 \) \( \Rightarrow \) \( V_L = 0 \) and \( L \) looks like a short.